

A Tutorial

Progress toward resolving Magnetic Reconnection Rate Problem

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S. Wang, L.-J. Chen et al. & MMS team

Outlines

- ★ Past —Present
- ★ MMS observations
- ★ Summary & Future (Unsolved questions)



Biased by my limited knowledge
& personal preference...

Past — Present

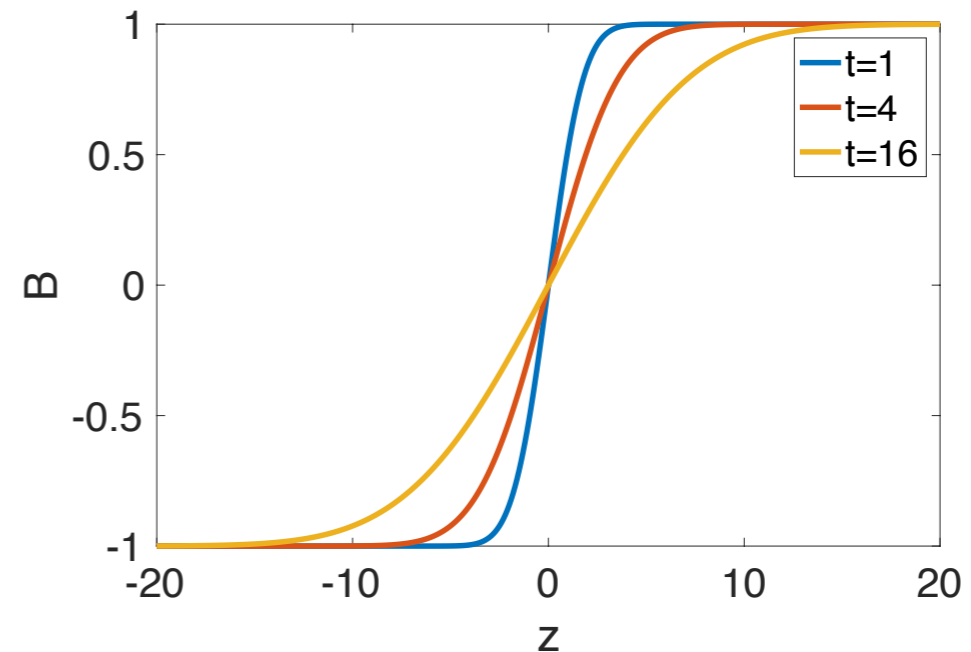
Magnetic Diffusion (<1953) (i.e., Dungey, 1953)



Induction eqn:

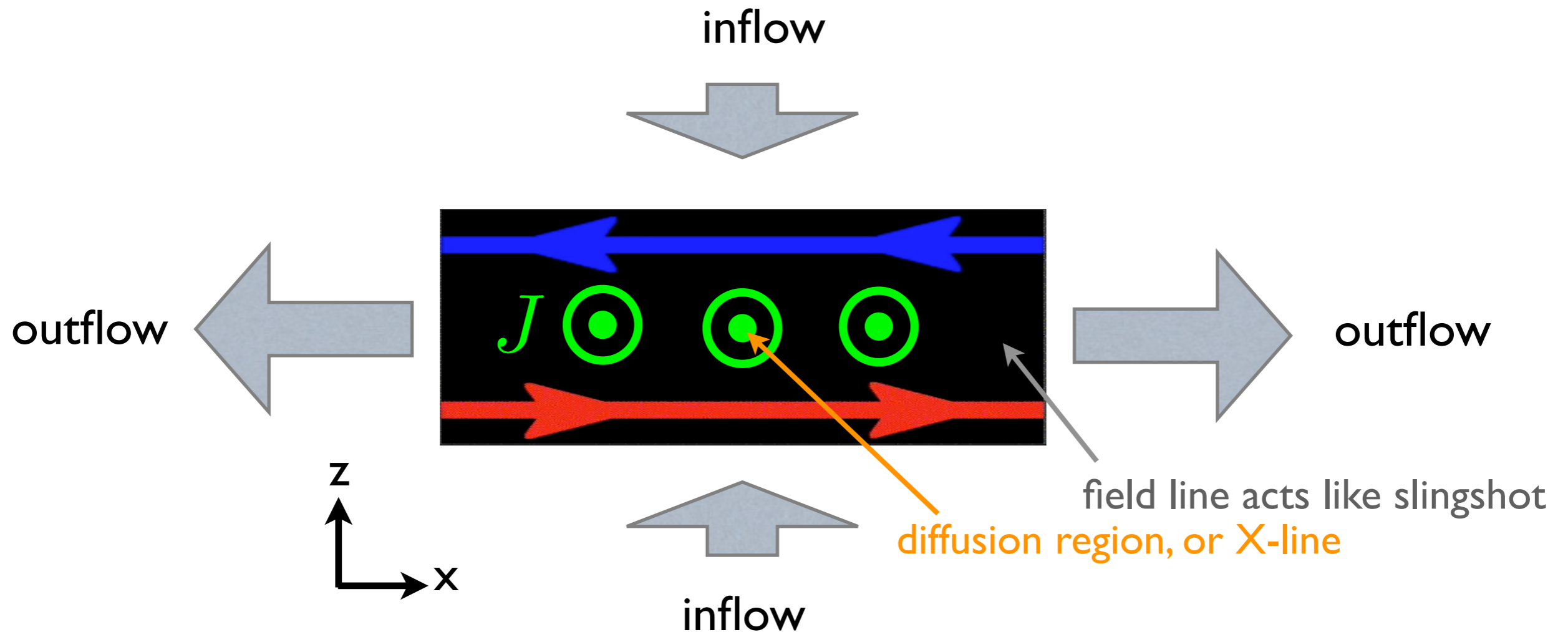
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

$$\rightarrow B = B_0 \operatorname{erf} \left[z / \sqrt{4\eta t} \right]$$



- Becomes broader & broader over time.... No steady state...
- Too slow to explain the dissipation of magnetic energy

Magnetic Reconnection



1. Inflow brings in magnetic flux (frozen-in)
2. Field lines break & reconnect (frozen-in is violated !!)
3. Reconnected field line shoots out plasma (frozen-in)
4. Pressure drop sucks in plasma inflow
1. Inflow brings in magnetic flux (frozen-in)
2.
3. ...

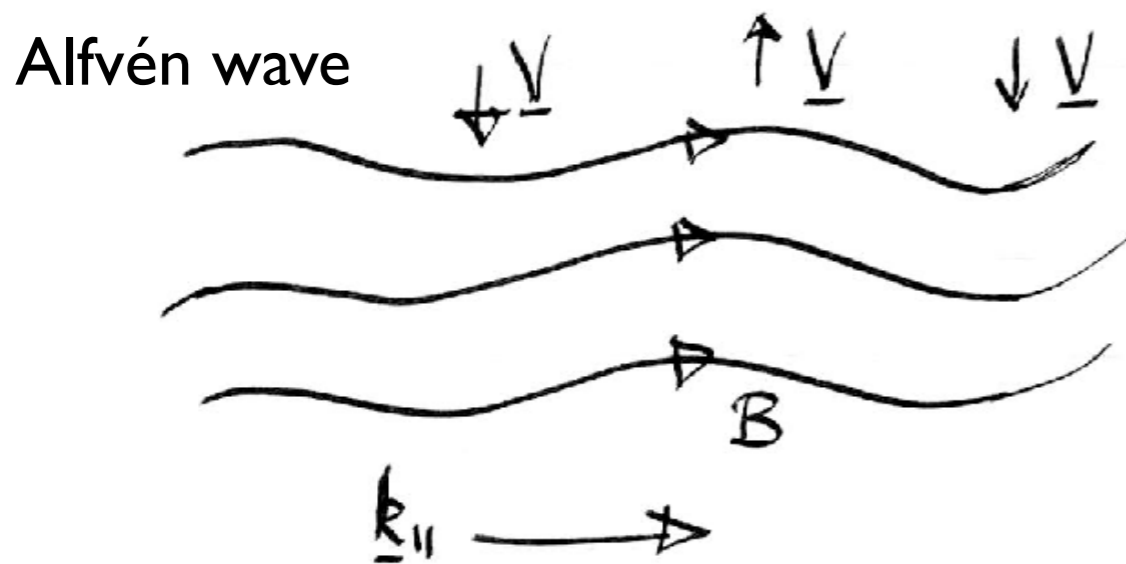
A self-driven process!!!

Magnetic tension & Alfvén waves

magnetic tension: $\frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi}$

$$\frac{B_y(\partial_y B_x)}{4\pi} \hat{x}$$

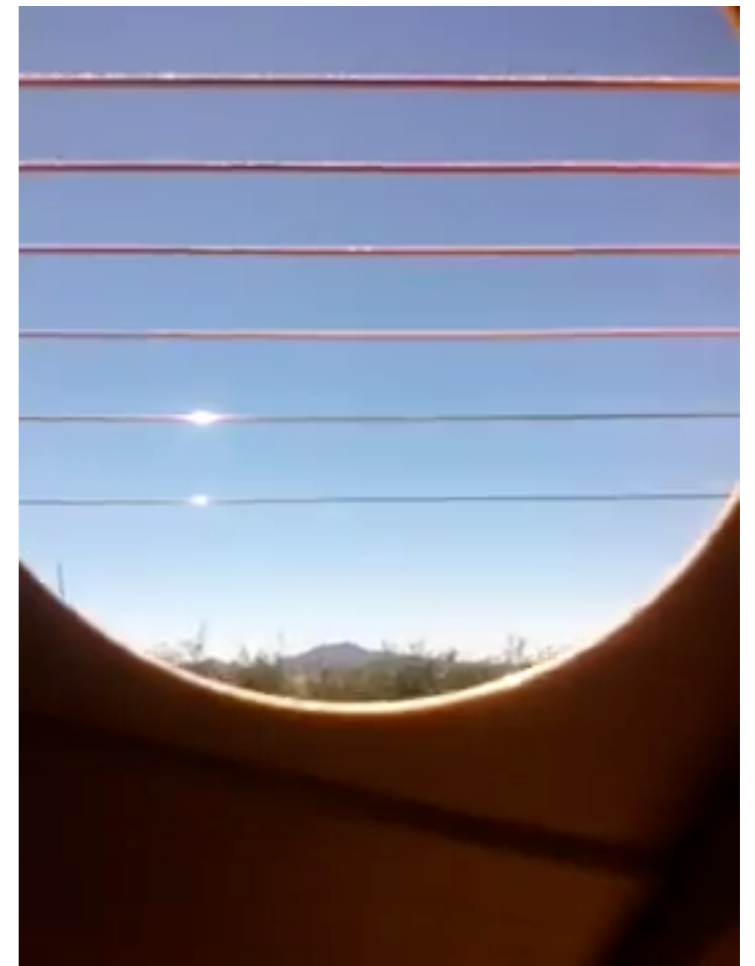
plasma inertia: $nm_i \frac{d\mathbf{V}}{dt}$



Alfvén speed

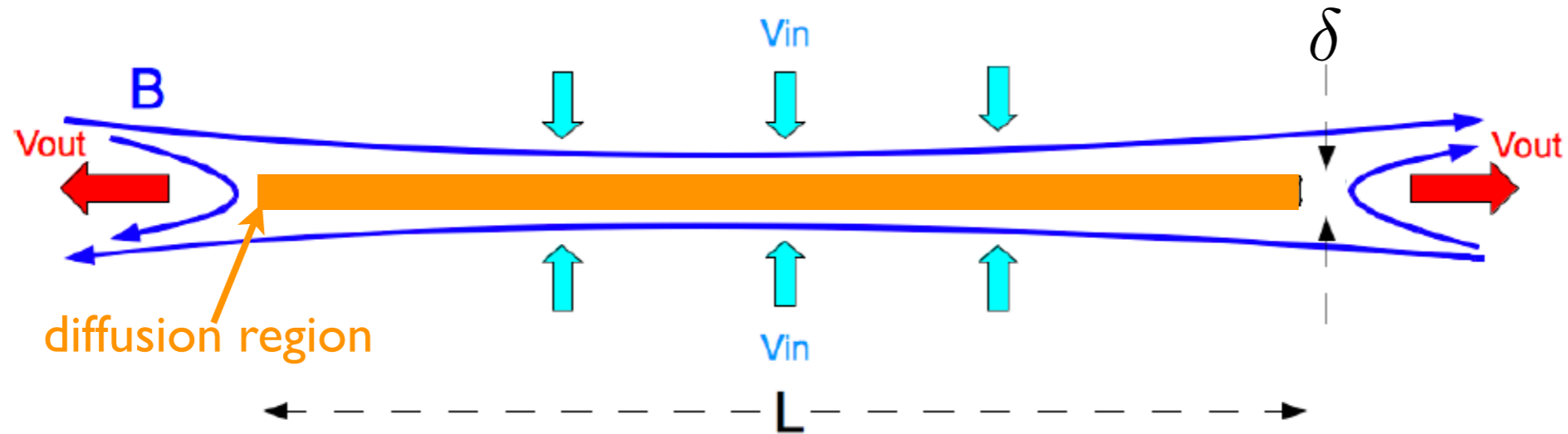
$$\rightarrow V_A \sim \frac{B}{\sqrt{4\pi nm_i}}$$

vibration of guitar strings



(Youtube: iphone 4 inside a guitar oscillation! VERY COOL!)

Sweet-Parker solution (1957)



mass conservation: $\nabla \cdot (n\mathbf{V}) \simeq 0 \quad \rightarrow V_{in}L \simeq V_{out}\delta$

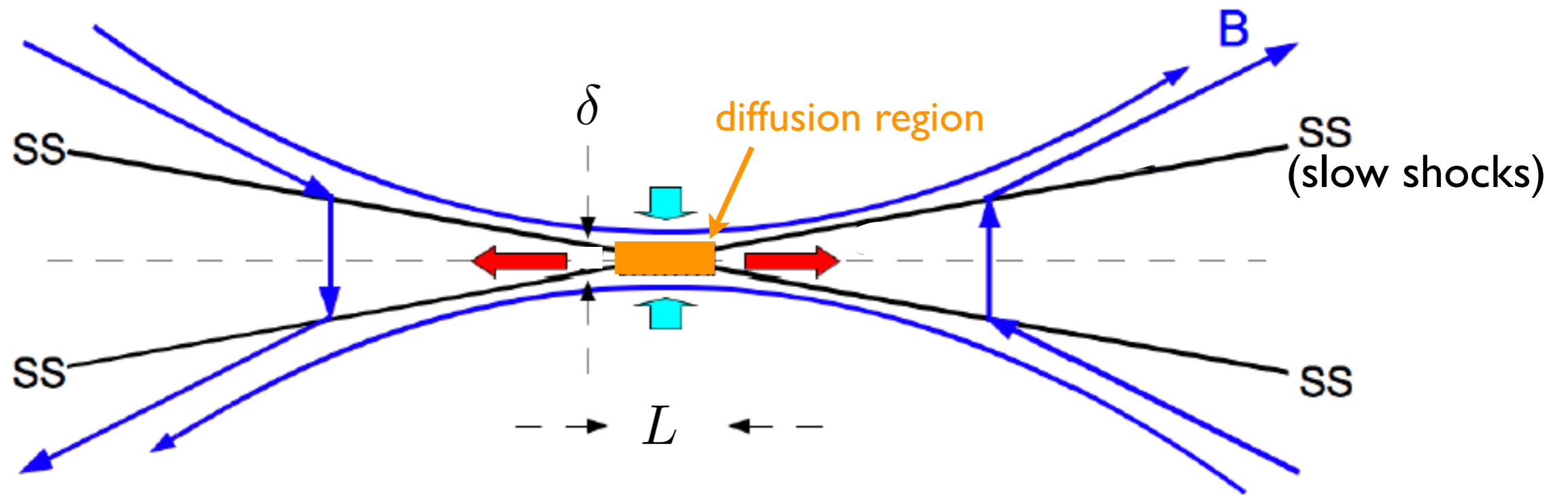
momentum eq.: $\frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi} \simeq nm_i \mathbf{V} \cdot \nabla \mathbf{V}$ $\rightarrow V_{out} \simeq \frac{B}{\sqrt{4\pi nm_i}} = V_A !$

tension inertia

normalized reconnection rate $\rightarrow R \equiv \frac{V_{in}}{V_A} \sim \frac{\delta}{L}$

- **However**, this model has a small δ/L , the rate is also too small to explain the time-scales in solar flare. (Parker 1963)
- To explain the flares, it requires $R \sim 0.1$. (Parker 1973)

Petschek solution (1964)

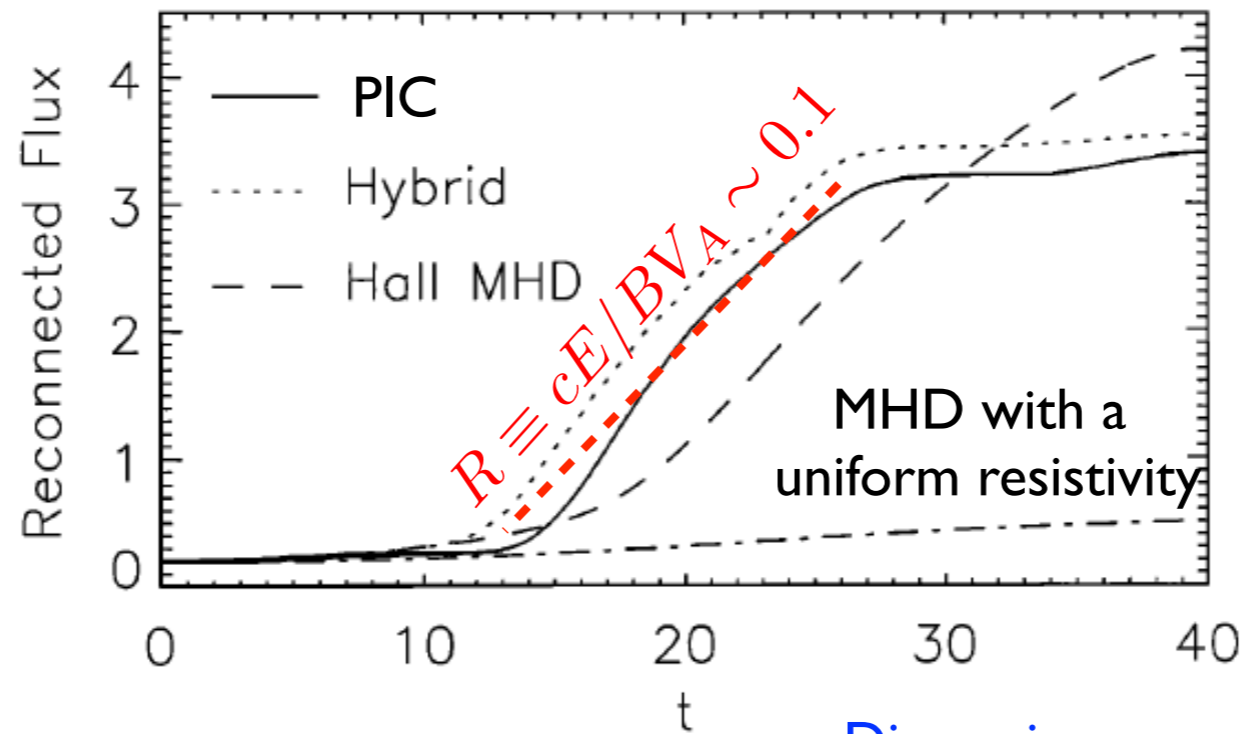


Reconnection rate is much larger if $R \sim \frac{\delta}{L} \uparrow$

- **However**, this is not a self-consistent solution. (Sato & Hayashi, 79; Biskamp, 86)
- In fact, standing switch-off slow shocks can hardly develop in fully kinetic simulations. (Liu+ 2011,2012)

*aspect ration \equiv aspect ratio of the diffusion region

GEM Reconnection Challenge (2001)



(Birn+, 2001)

Dispersive wave picture

(Sonnerup 79; Shay+ 98; Rogers+ 01; Drake+ 08)

Ohm's Law in plasmas:

$$\mathbf{E} + \frac{\mathbf{v}_i \times \mathbf{B}}{c} = \eta \mathbf{J} + \frac{1}{ne} \mathbf{J} \times \mathbf{B} - \frac{1}{ne} \nabla \cdot \mathbf{P}_e + \frac{m_e}{e^2} \frac{d\mathbf{J}}{dt}$$

	convection	resis.	Hall term	pressure	inertia	
MHD:	✓					slow
Hall MHD:	✓		✓			↑ fast with $R \sim 0.1$ ↓
Hybrid:	✓		✓		✓	
PIC:			✓	✓	✓	

Standing Dispersive Wave Picture

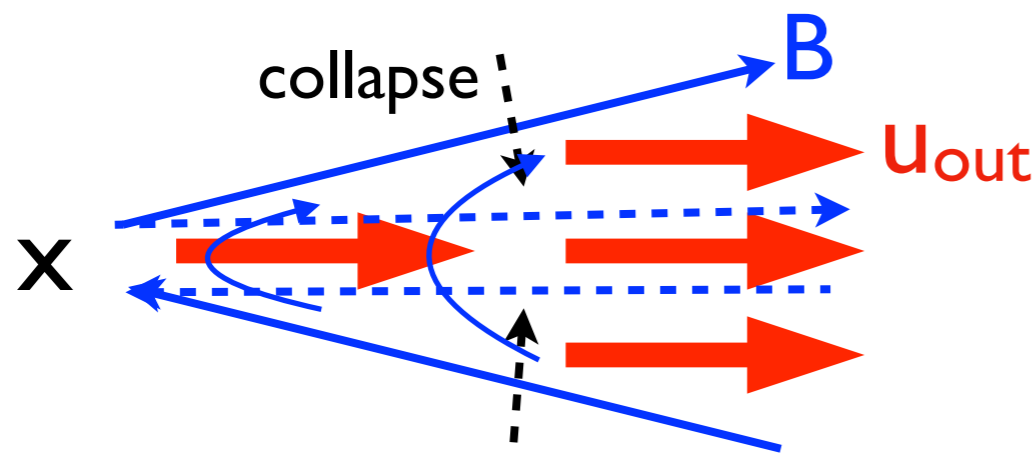
— outflow is driven by magnetic tension force

Without the Hall term...

Alfvén wave

$$\omega \propto k$$

$$\rightarrow u_{out} \sim \omega/k \sim \text{constant}$$



— collapses back to a long Sweet-Parker layer

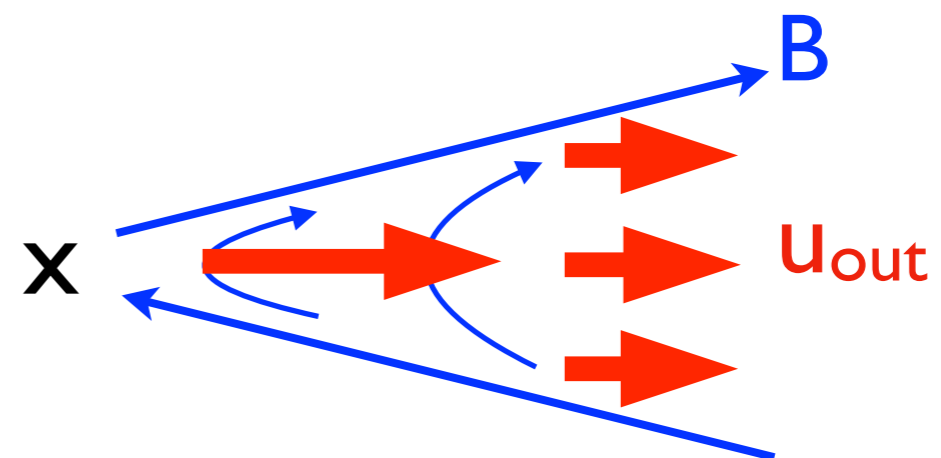
With the Hall term...

Whistler wave $(b_g = 0)$

Kinetic Alfvén wave $(b_g \neq 0)$

$$\omega \propto k^2$$

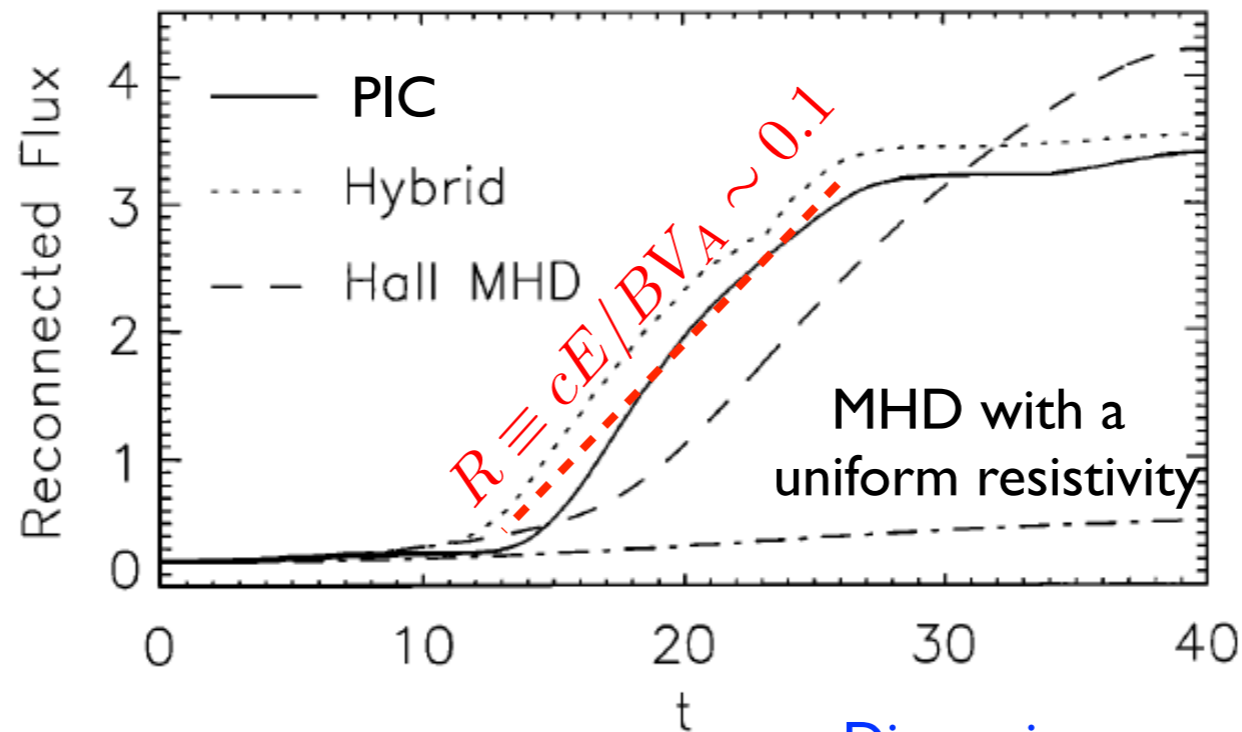
$$\rightarrow u_{out} \sim \omega/k \propto k$$



— stays opened!

- This seems to explain the difference of reconnections in resistive-MHD vs. Two-fluid/Hybrid/PIC models. (Birn+ 2001, Rogers+ 2001, Shay+ 1998, Mandt+ 1994)

GEM Reconnection Challenge (2001)



(Birn+ 2001)

Dispersive wave picture

(Sonnerup 79; Shay+ 98; Rogers+ 01; Drake+ 08)

Ohm's Law in plasmas:

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	convection	resis.	Hall term	pressure	inertia	
MHD:	✓					slow
Hall MHD:	✓		✓			↑ fast with $R \sim 0.1$
Hybrid:	✓		✓		✓	
PIC:			✓	✓	✓	↓

However, electron-positron (PIC):

(Bessho & Bhattacharjee, 05; Daughton+ 07; Swisdak+ 08; Liu+ 09)

strong guide field limit (PIC):

(Liu+14; TenBarge+14; Stainer+15; Cassak+15)

also fast with $R \sim 0.1$

Q1: Why is the fast rate $R \sim 0.1$?

Q2: What is the localization mech.?

To be solved.

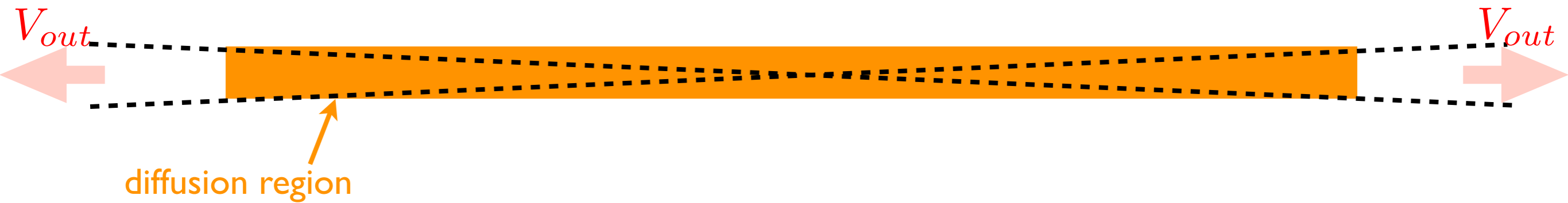
Q1: How to explain the fast reconnection rate value of order 0.1
in different systems?

-- including PIC, hybrid, Hall-MHD, MHD with a localized resistivity...etc

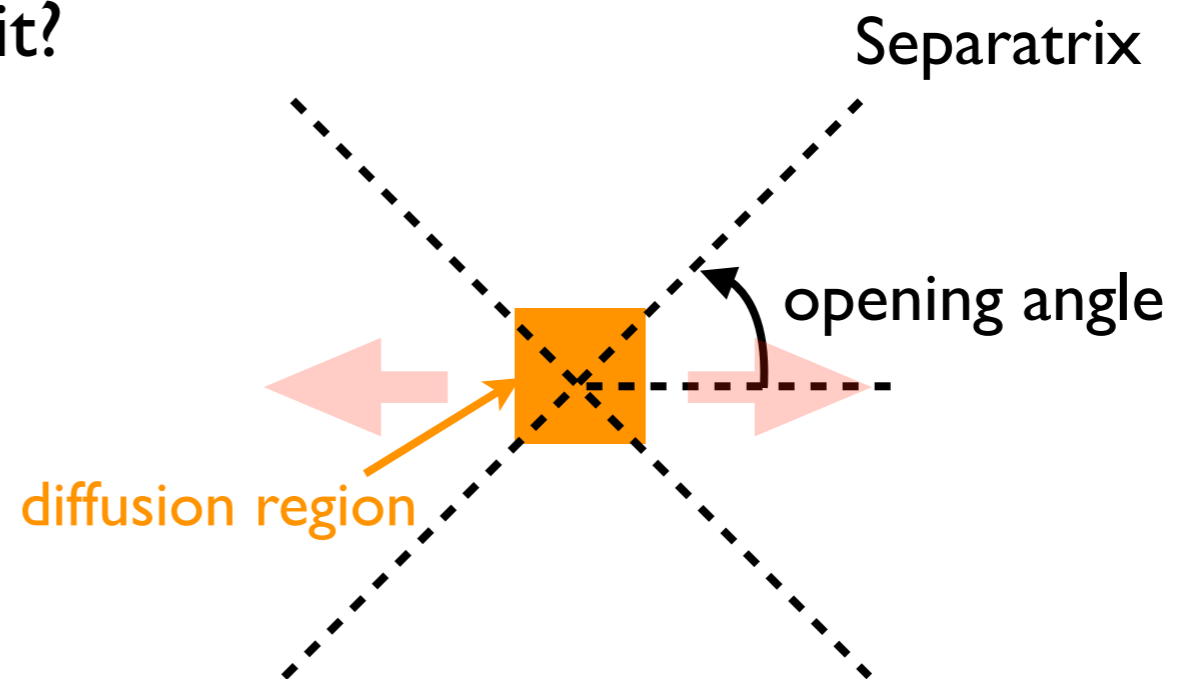
*clue: can not be the diffusion-scale physics!

Two extreme limits...

In the small δ/L limit, $R \sim \delta/L \rightarrow 0$



How about the large δ/L limit?



It turns out that when $\delta/L \rightarrow 1$, $R \rightarrow 0$!

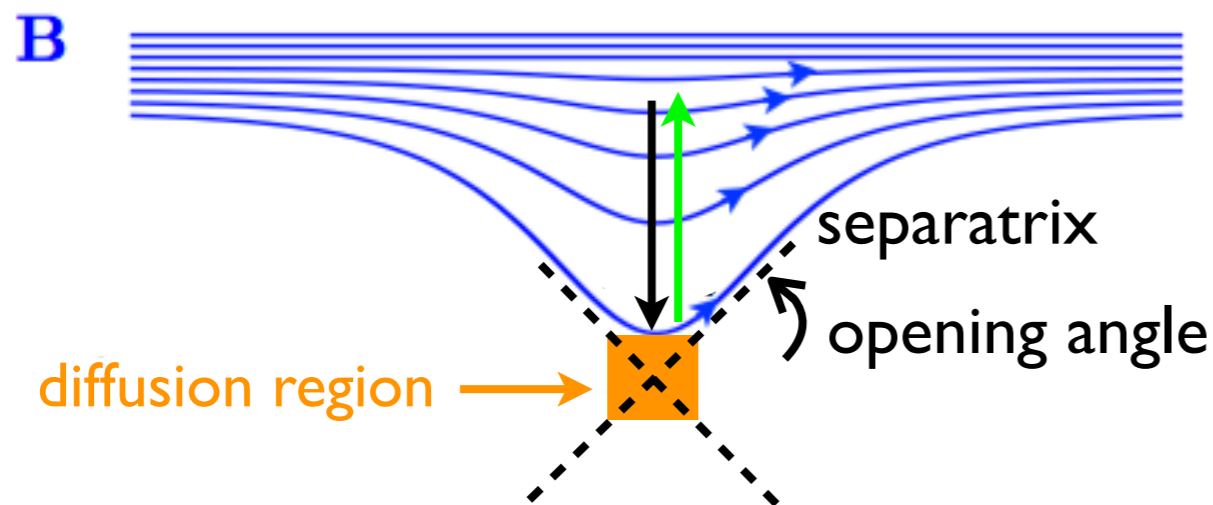
◆ There must be a maximum in between these two limits~

The Key: Geometry & Force balance!

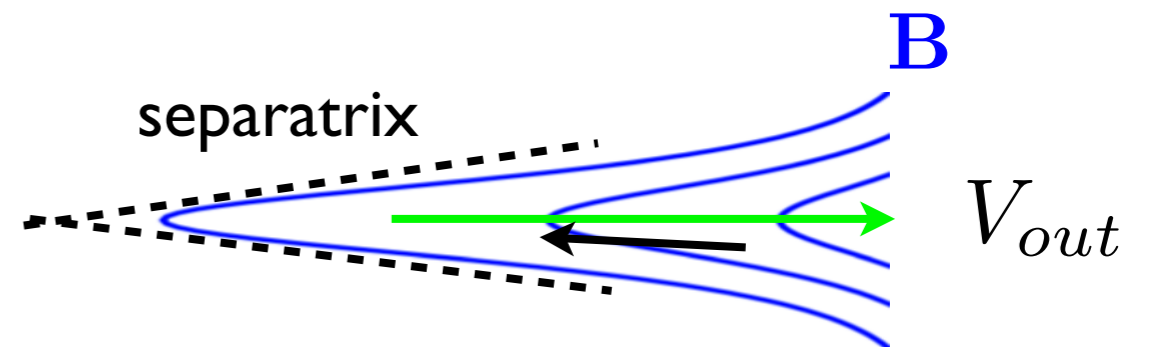
In the large diffusion region aspect ratio, δ/L , limit

$$\frac{\text{tension}}{4\pi} \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi} \simeq \frac{\text{magnetic pressure}}{8\pi} \frac{\nabla(B^2)}{8\pi} + \text{inertia} \quad + \quad nm_i \mathbf{V} \cdot \nabla \mathbf{V}$$

@ inflow region



@ outflow region



→ reduction of the reconnecting field!!!

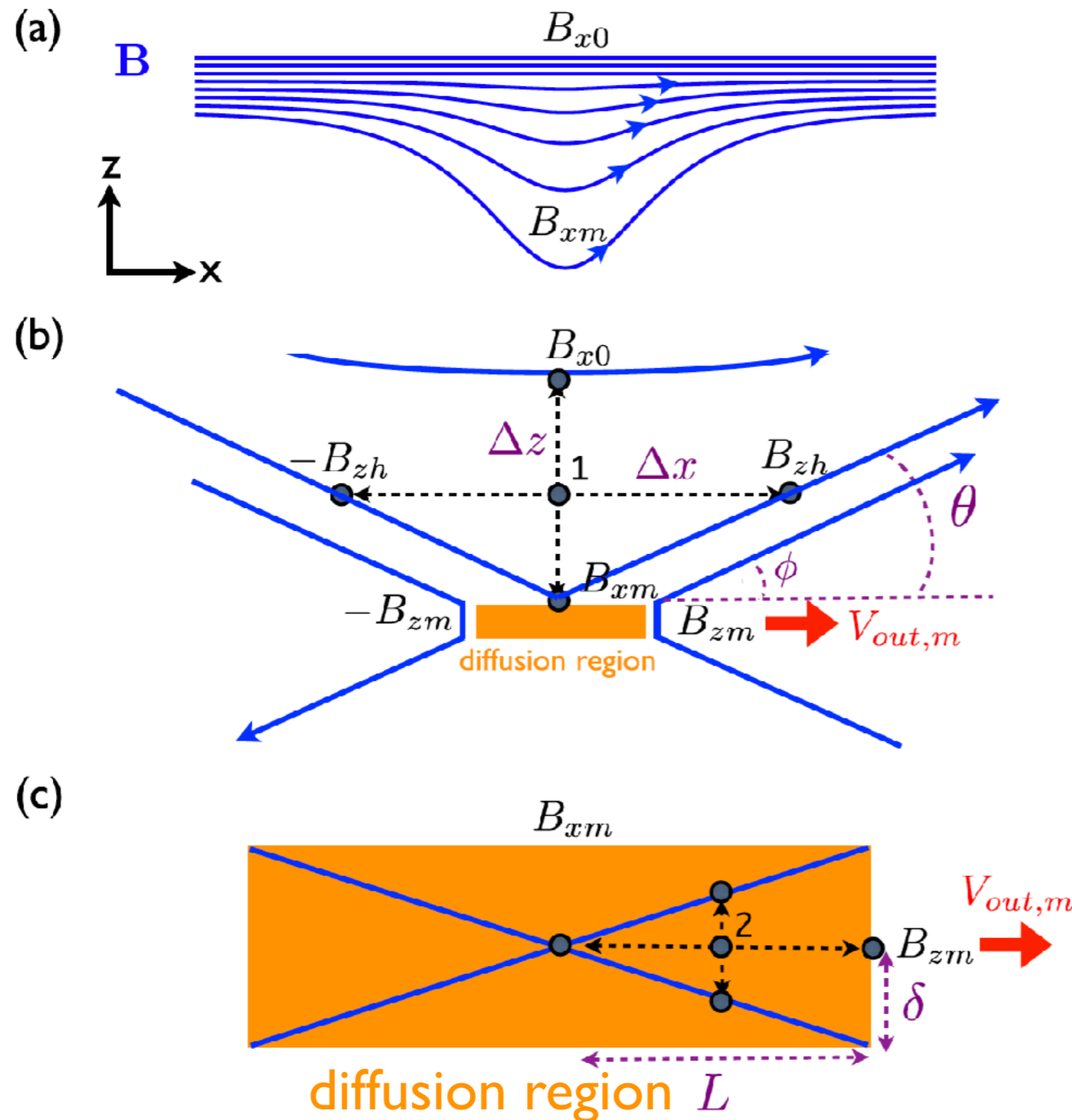
→ $R \downarrow$

→ reduction of the outflow speed!!!

→ $R \downarrow$

- Constraints imposed at the inflow & outflow regions (upper) bound the rate!

Back-of-the-envelope calculation...



Introduce the scale-separation~

analyze the force-balance at point 1

$$\rightarrow B_{zm}(S)$$

analyze the force-balance at point 2

$$\rightarrow V_{out,m}(S)$$

Connecting these two regions to get the rate~

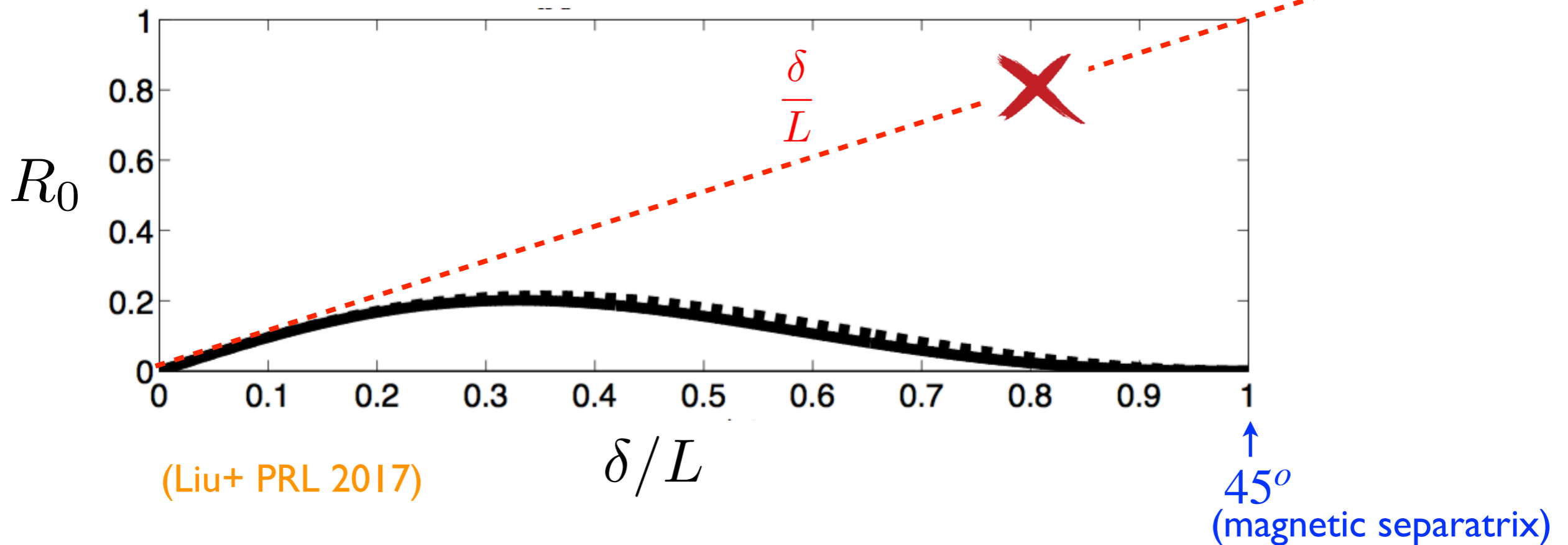
$$E_y = B_{zm} V_{out,m} / c$$

Explanation of the fast rate ~ 0.1

-- Geometrical consideration!

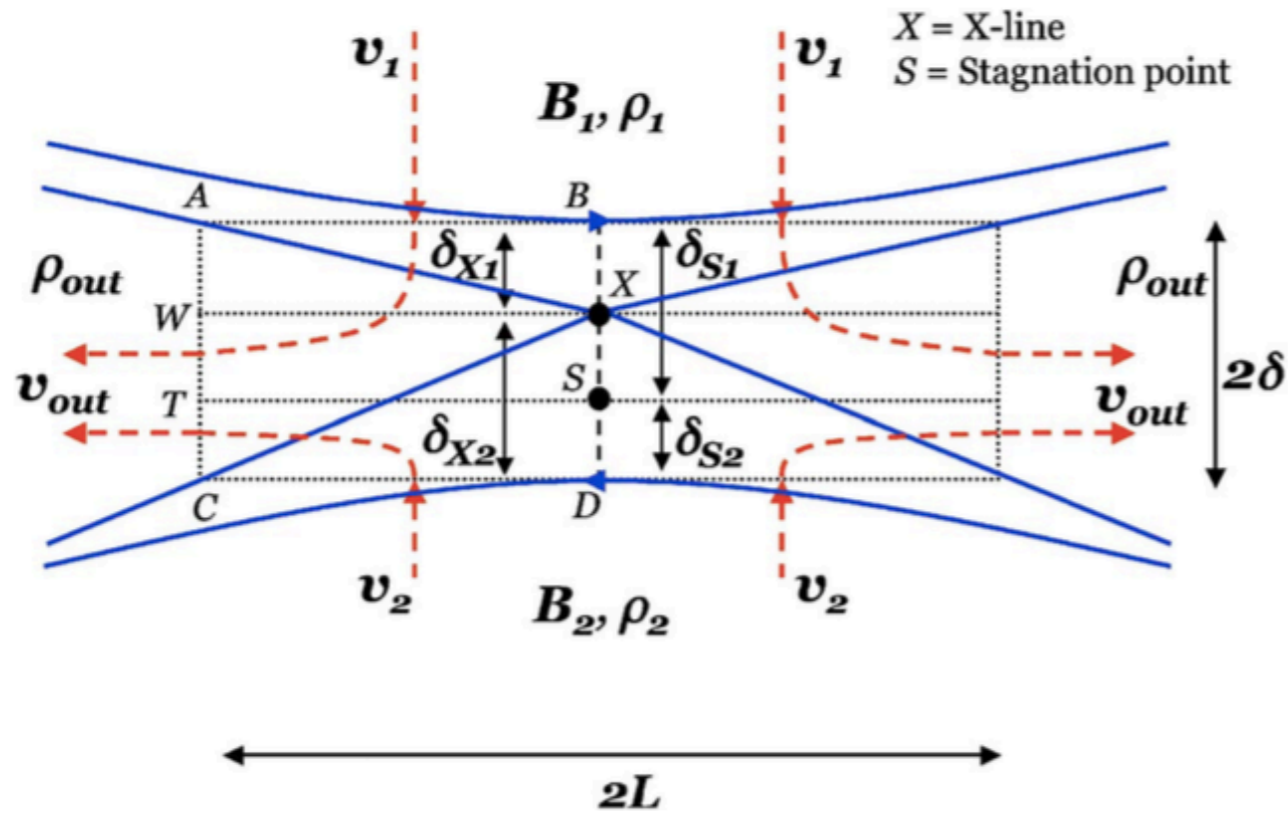
$$R_0 \equiv \frac{cE_y}{B_{x0}V_{A0}} = \left(\frac{B_{zm}}{B_{x0}}\right) \left(\frac{V_{out,m}}{V_{A0}}\right) \simeq \frac{\delta}{L} \left[\frac{1 - (\delta/L)^2}{1 + (\delta/L)^2} \right]^2 \sqrt{1 - \left(\frac{\delta}{L}\right)^2}$$

reduction of V_{out}
reduction of reconnecting B



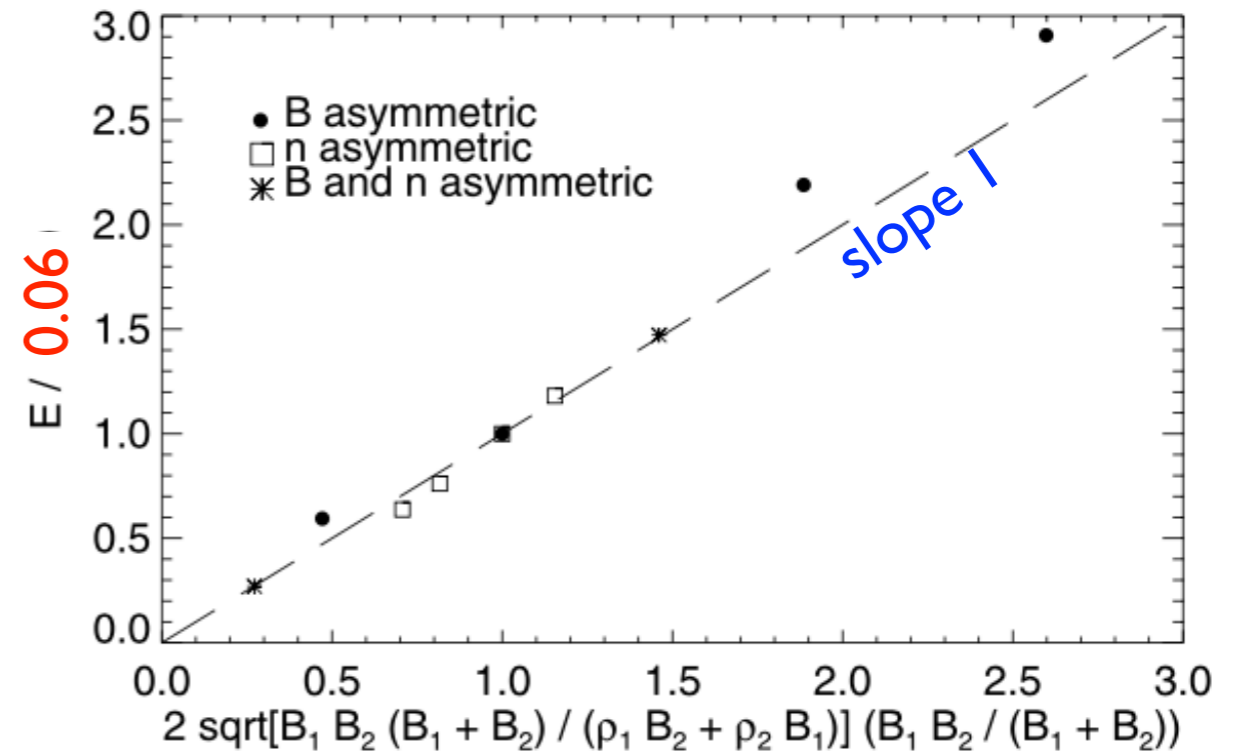
- Fast rate $R \sim O(0.1)$ is an upper bound value.
- Reconnection tends to proceed near the most efficient state, which has $R \sim O(0.1)$.
- Nicely, rate is insensitive to δ/L near this state.

Asymmetric Reconnection



Two-fluid simulations

Reconnection Rate



Cassak & Shay GRL (2008)

Cassak-Shay formula

$$E_{CS} = 2 \left(\frac{B_1 B_2}{B_1 + B_2} \right) \left(\frac{V_{out}}{c} \right) \left(\frac{\delta}{L} \right)_{eff}$$

Cassak & Shay, PoP (2007)

where

$$\frac{V_{out}}{c} = \sqrt{\frac{B_1 B_2}{4\pi} \left(\frac{B_1 + B_2}{B_1 \rho_2 + B_2 \rho_1} \right)}$$

Swisdak & Drake, GRL (2007)

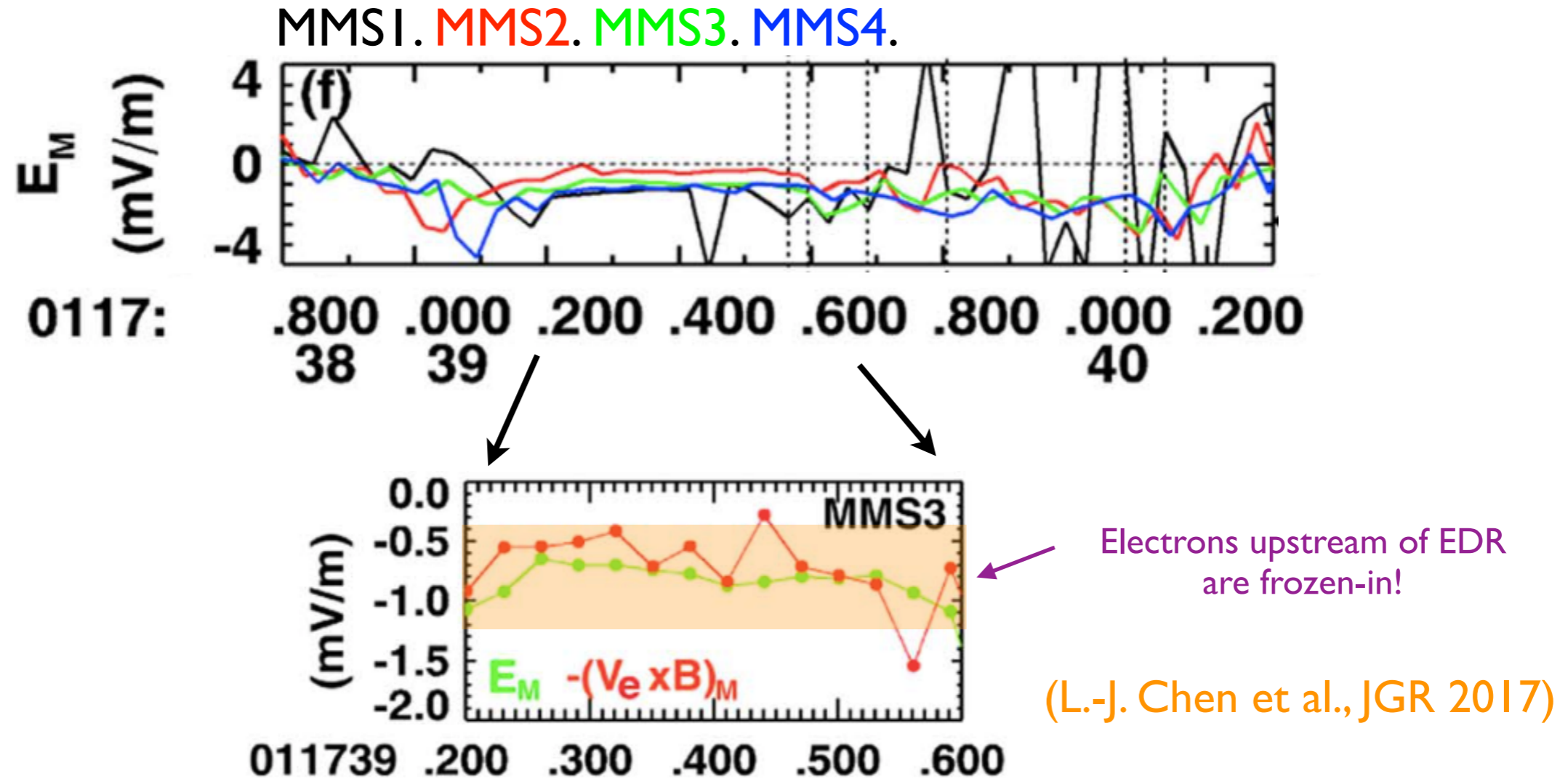
and $(\delta/L)_{eff} \simeq 0.1$

Liu+ GRL (2018)

MMS observations

MMS observations

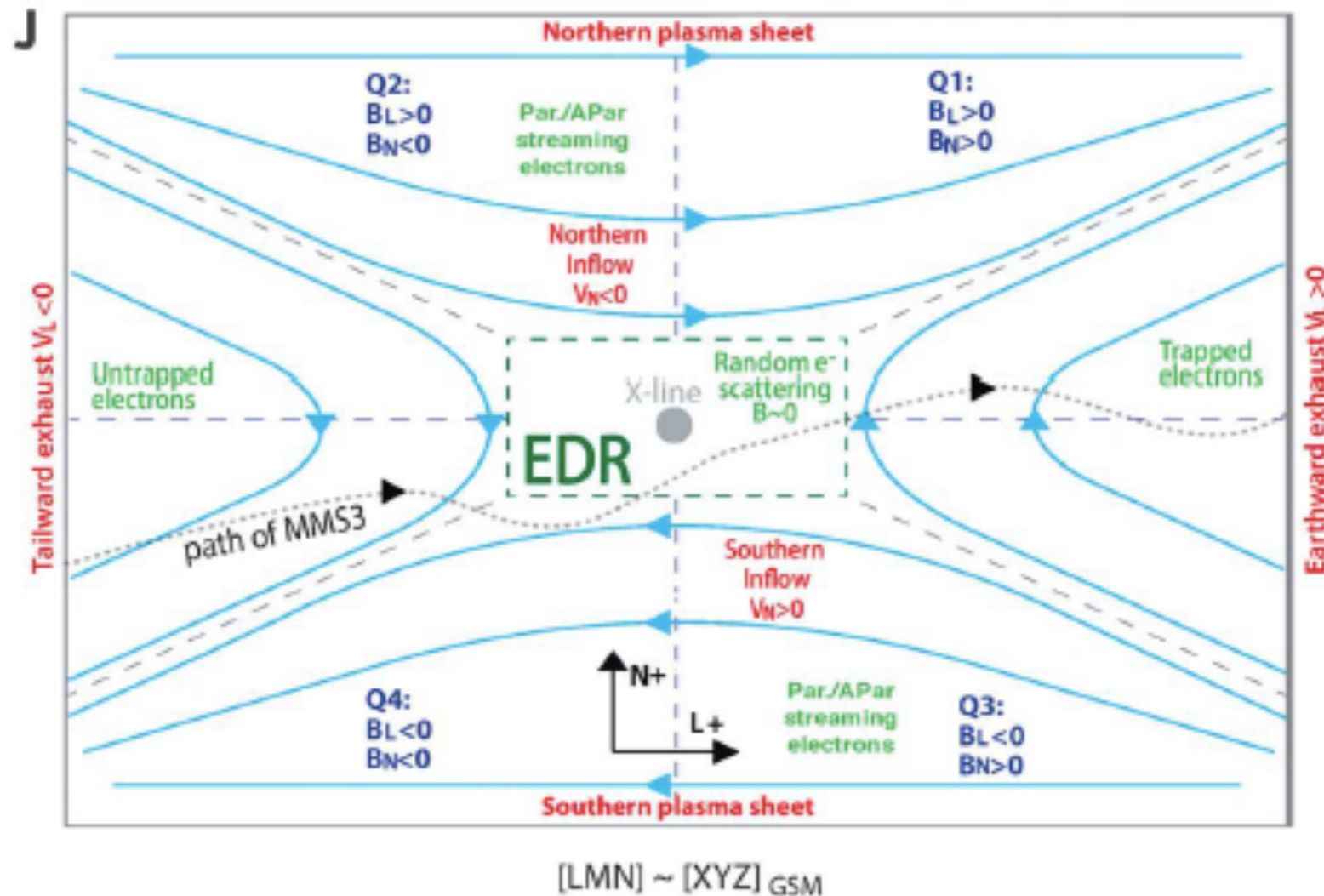
December 14, 2015 event: $B_g \sim 0.2$, $B_{L2}/B_{L1} \sim 1.3$, $n_2/n_1 \sim 6.8$



- An uniform electric field over at least 8 electron skin depths corresponds to a normalized rate ~ 0.1 .
- The rate of the October 16, 2015 event was estimated to be ~ 0.3 . (Burch+ Science 2016)

MMS observations

Torbert' 7/11 event

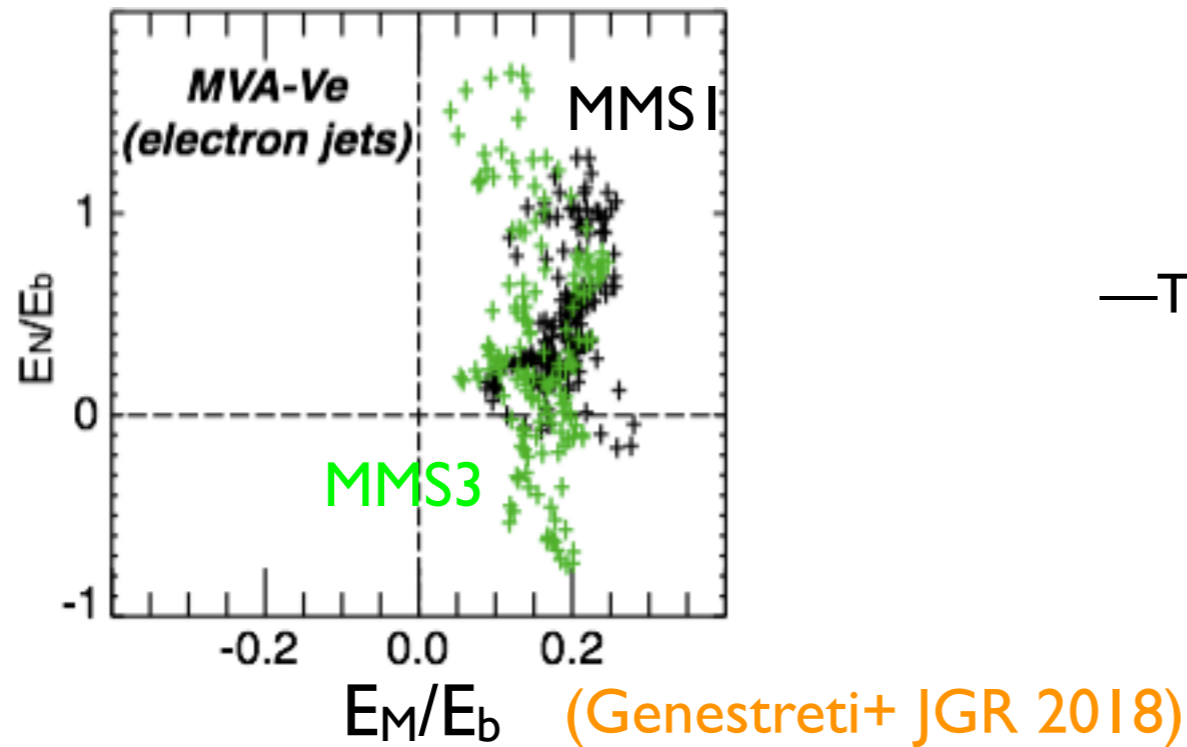


(Torbert+ Science 2018)

- Measuring the aspect ratio of EDR $\sim 0.1-0.2$
 - Using timing analysis to get L.
 - Using current density to get δ .

MMS observations

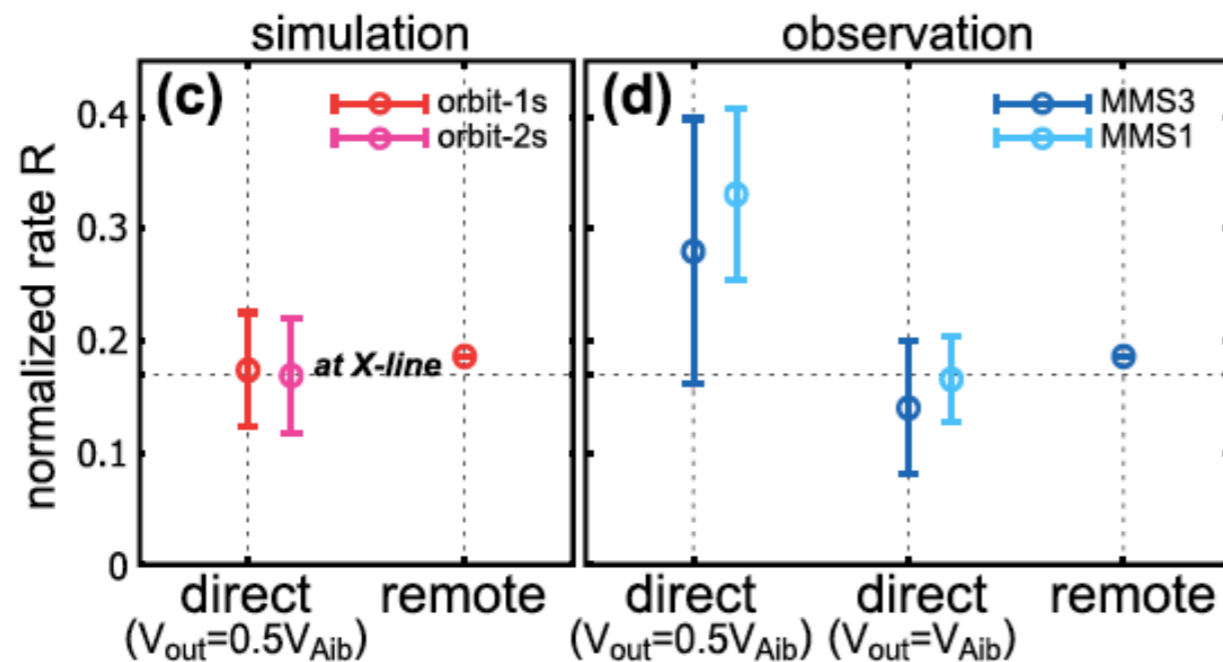
7/11 event



Measuring E_M

—Tried 14 different LMN coordinate systems

$$R \sim 0.18 \pm 0.035$$



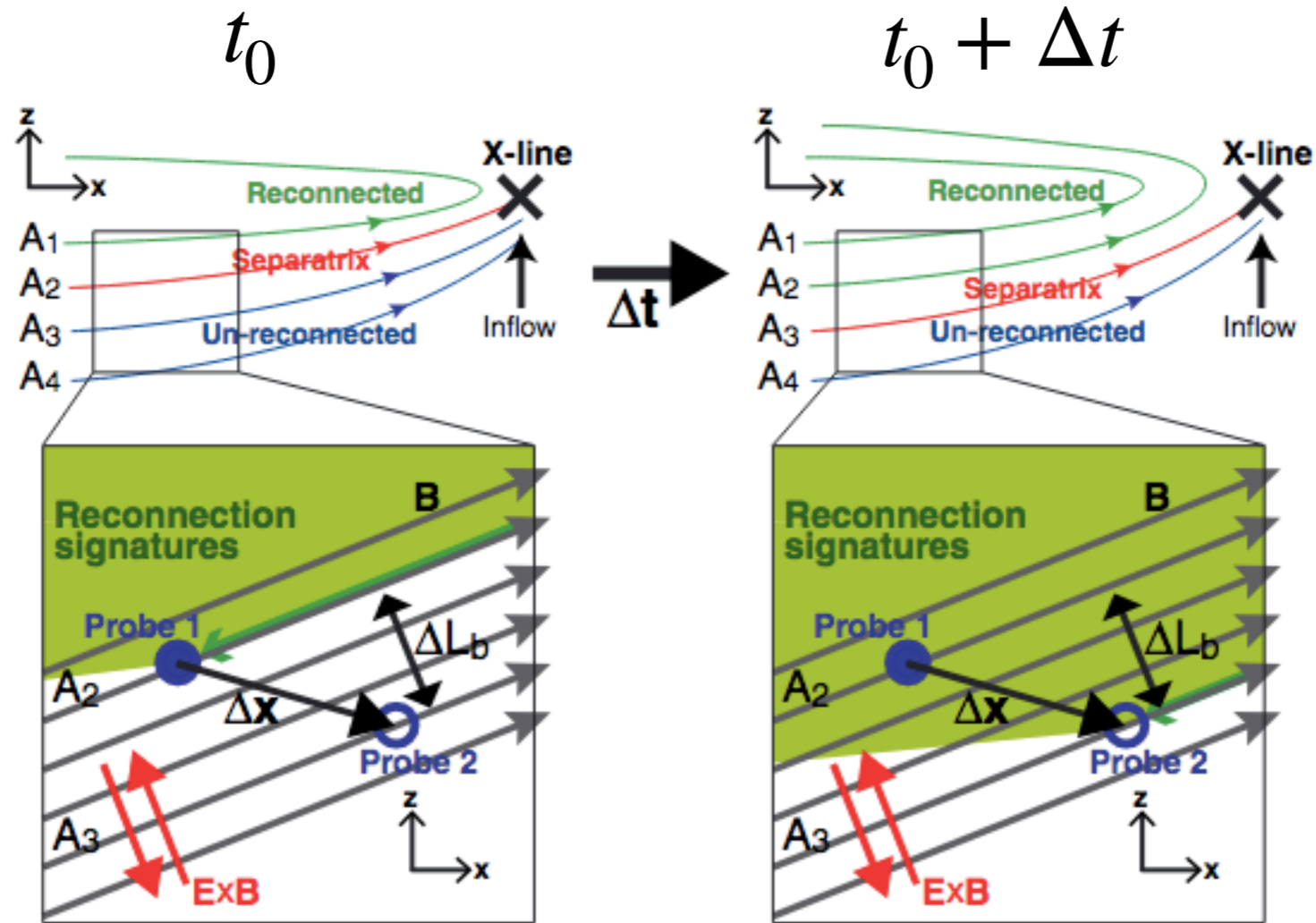
Measuring E_M

—Took advantage of the close comparison with 2D PIC simulations~

(Nakamura+ JGR 2018)

MMS observations

— new technique in measuring the rate



(Nakamura+ JGR 2018)

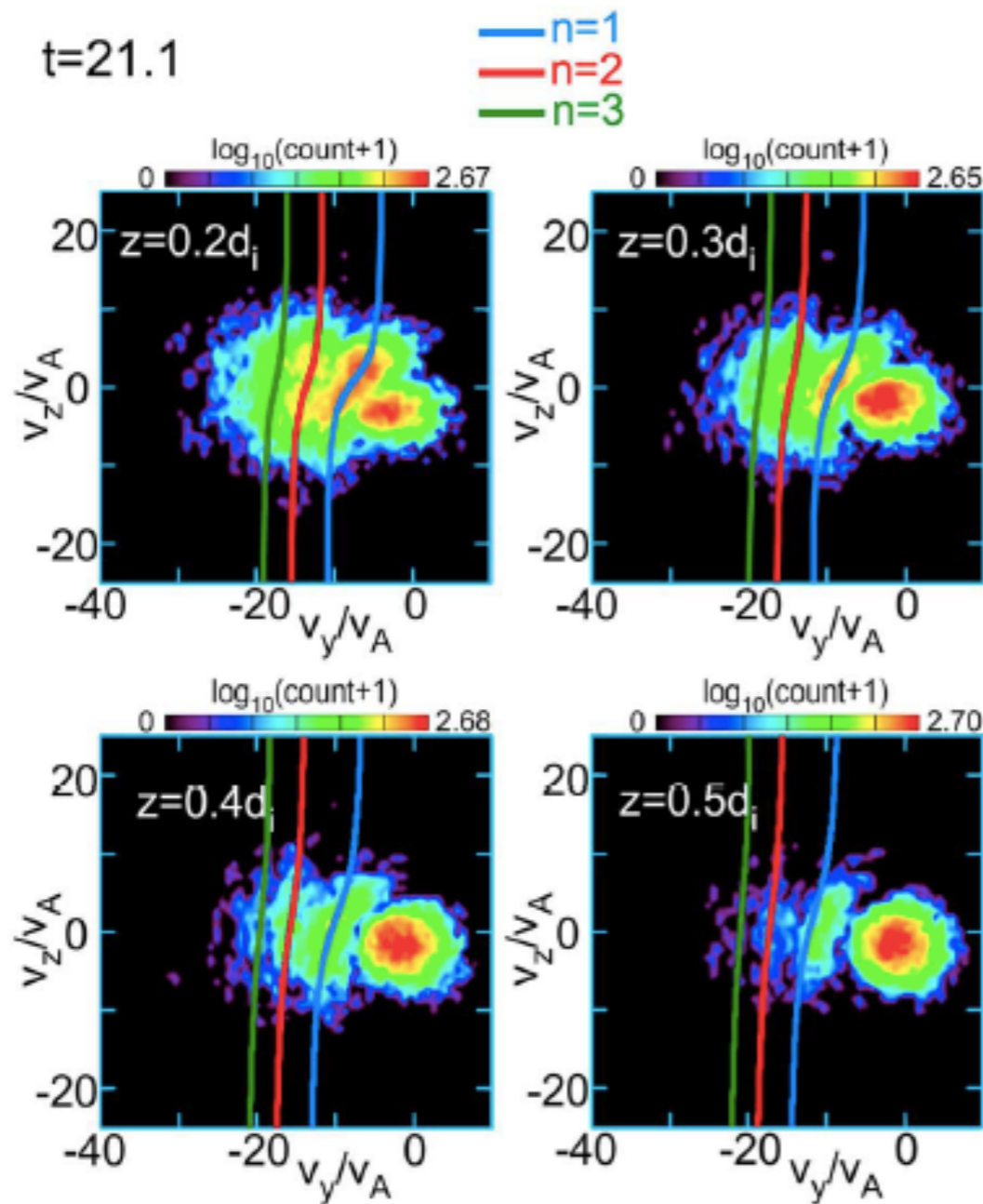
$$E_r \sim -\frac{(\Delta \mathbf{X} \times \mathbf{B})_y}{\Delta t} \sim -[\mathbf{v}_{\text{tim}} - \mathbf{v}_c] \times \mathbf{B}_y$$

convection of the magnetic flux
respect to the probe

- Measuring the flux difference at separatrix to infer the reconnection rate remotely!

MMS observations

— new technique in measuring the rate



$$b = dB_x/dz$$

$$k = dE_z/dz$$

$$\Delta v_y = \left[\left(-v_{y0} - \frac{ck}{b} \right)^{3/2} + \frac{3\pi}{4} (2n+1) \left(\frac{mc}{eb} \right)^{1/2} \frac{eE_r}{m} \right]^{2/3} - \left[\left(-v_{y0} - \frac{ck}{b} \right)^{3/2} + \frac{3\pi}{4} (2n-1) \left(\frac{mc}{eb} \right)^{1/2} \frac{eE_r}{m} \right]^{2/3}$$

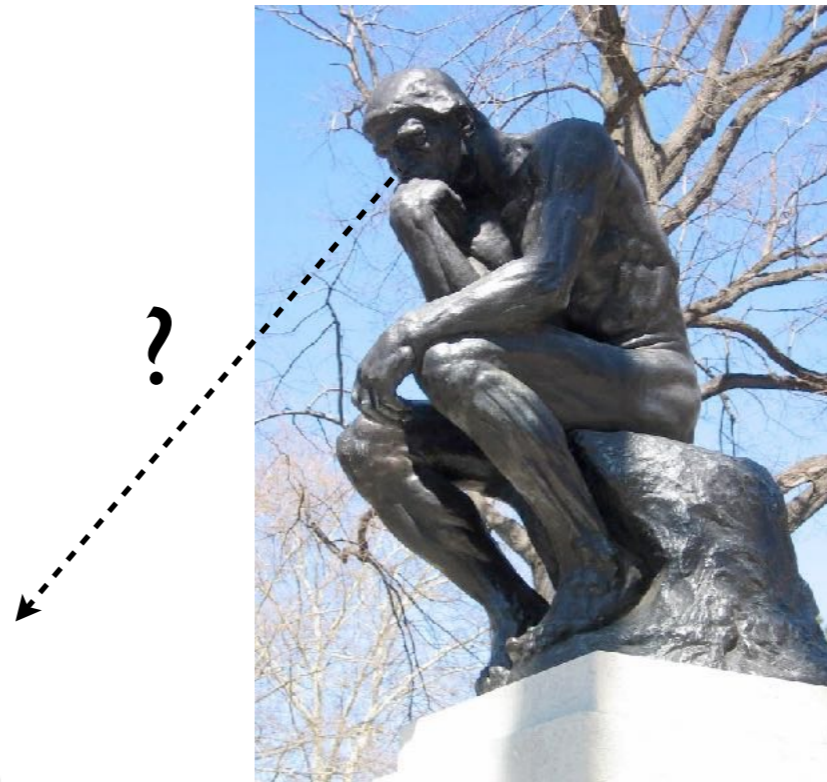
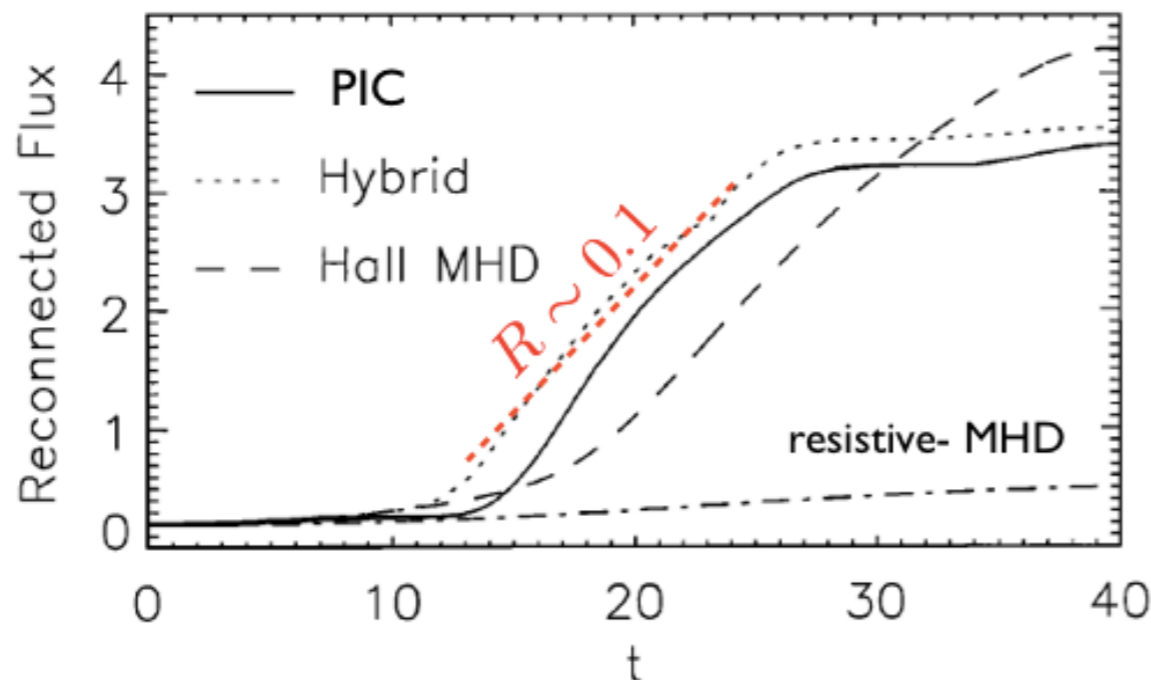
(Bessho+ GRL 2018)

- Inferring reconnection rate from particle distributions at the diffusion region.
 - E_R accelerates electrons in the out-of-plane (-y) direction.
 - $R \sim 0.22-0.28$ for the 7/11 event.

Summary & future (unsolved questions)

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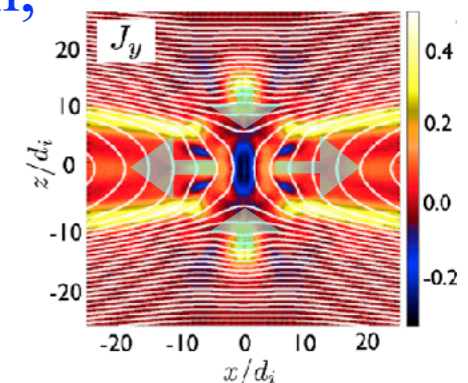
- ★ 0.1 is an upper bound value.



- ★ What is the localization mechanism in the standard regime?

- Why is MHD with an uniform resistivity so different? (the only exception.?)
- While a localization mechanism is needed for fast reconnection, different systems may have different localization mechanisms.

(Liu+, PoP, 2018)

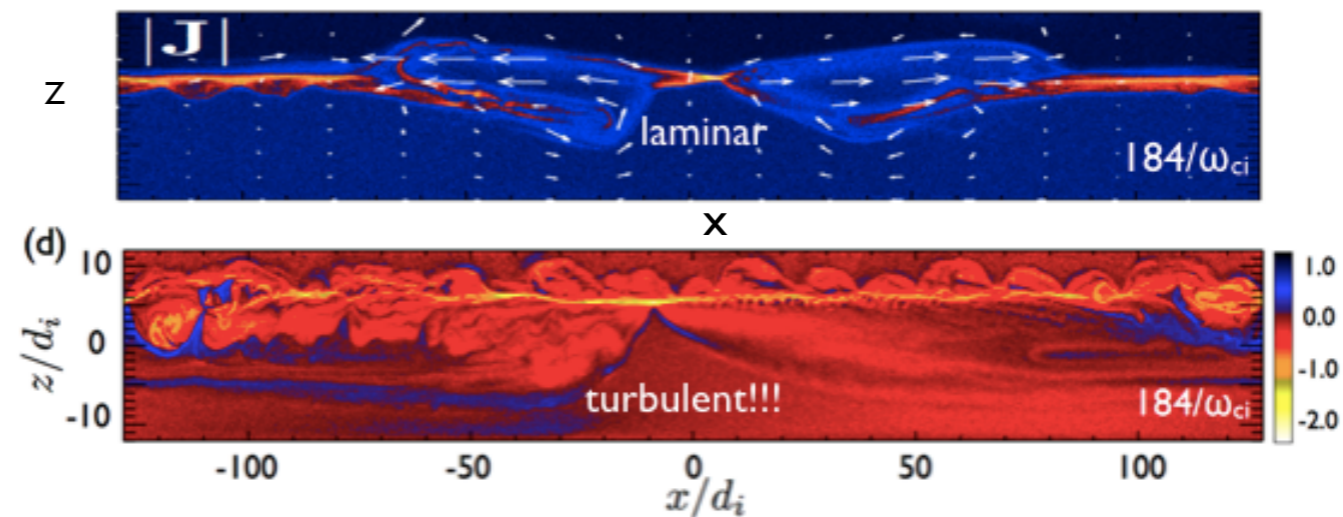
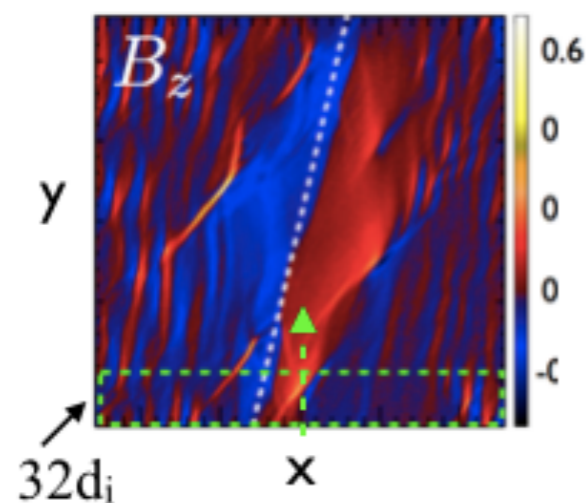


Summary & future (unsolved questions)

- ★ Turbulence!? if yes, how does it affect reconnection rate?
 - and how to measure the rate in a turbulent sheet using MMS???

p.s. be cautious about the periodic boundary condition in small simulations.

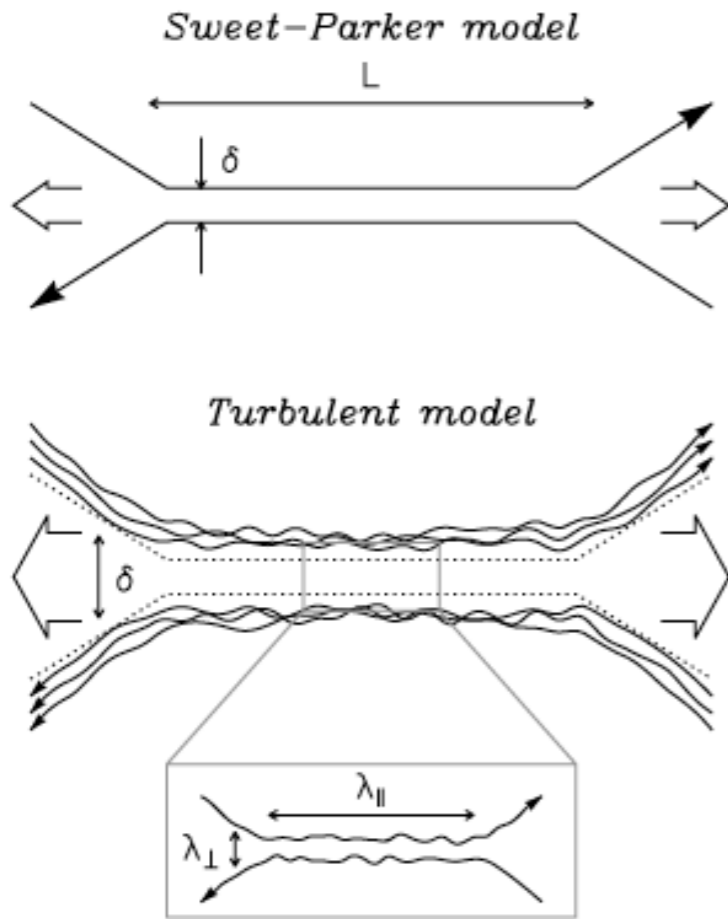
(Liu+ JGR 2018)



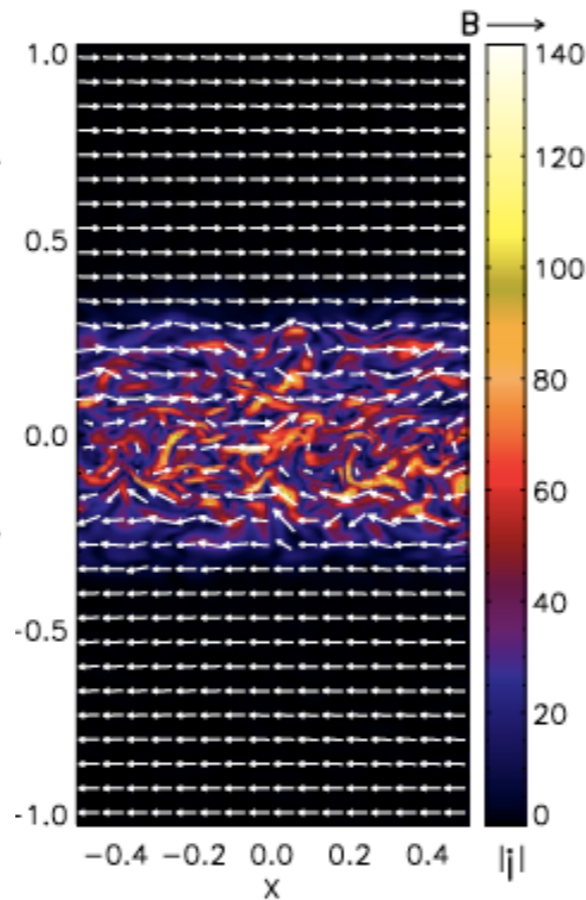
Large box

Small box

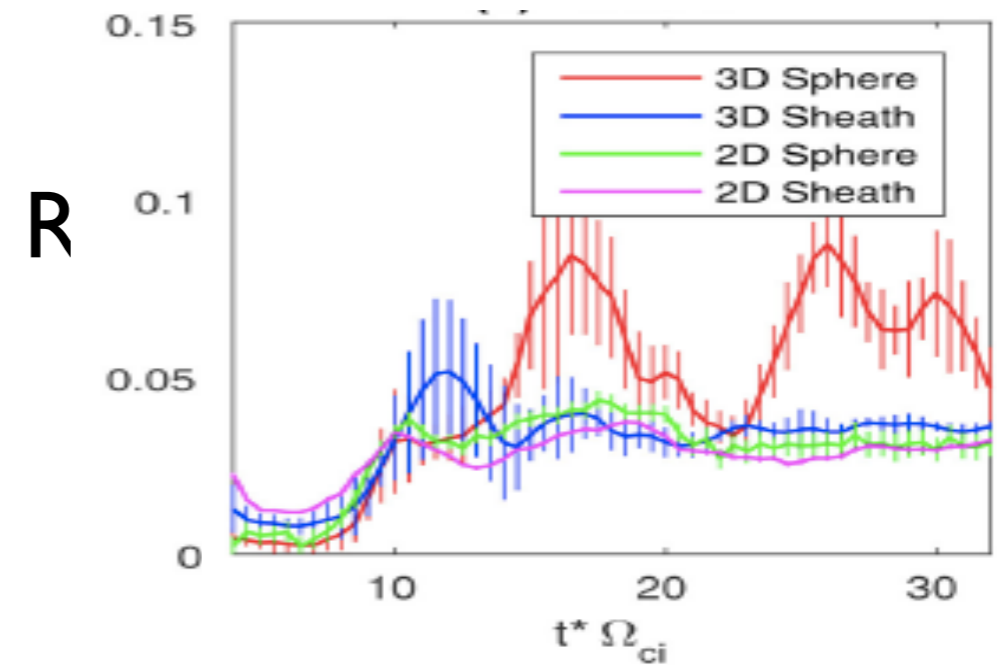
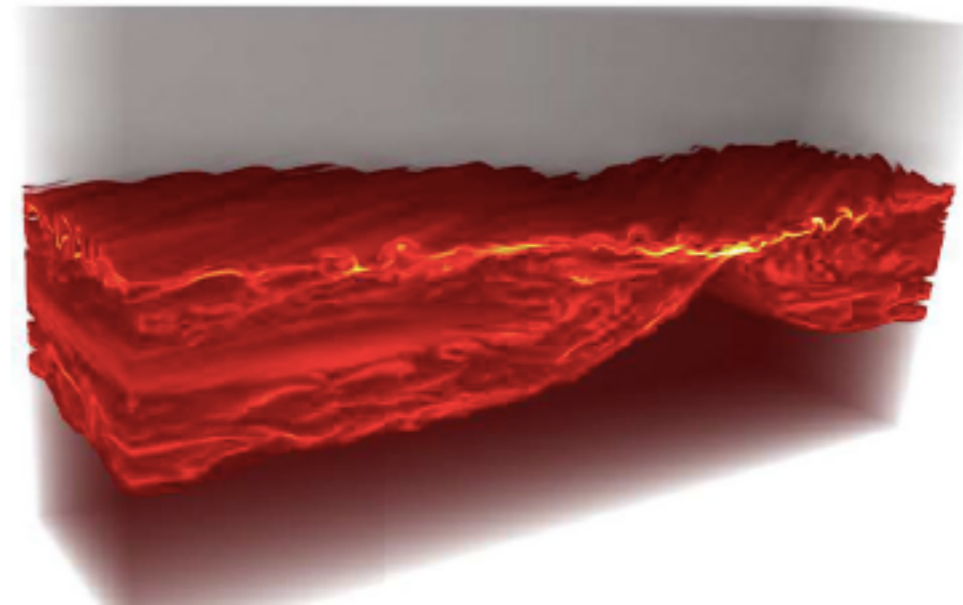
Turbulence



resistive-MHD
+ external driving



PIC- asymmetric



$$V_{\text{rec}} = V_A \min \left[\left(\frac{L}{l} \right)^{1/2}, \left(\frac{l}{L} \right)^{1/2} \right] \left(\frac{v_l}{V_A} \right)^2$$

(Kowal+ APJ 2009,
Lazarian & Vishniac, 1999)

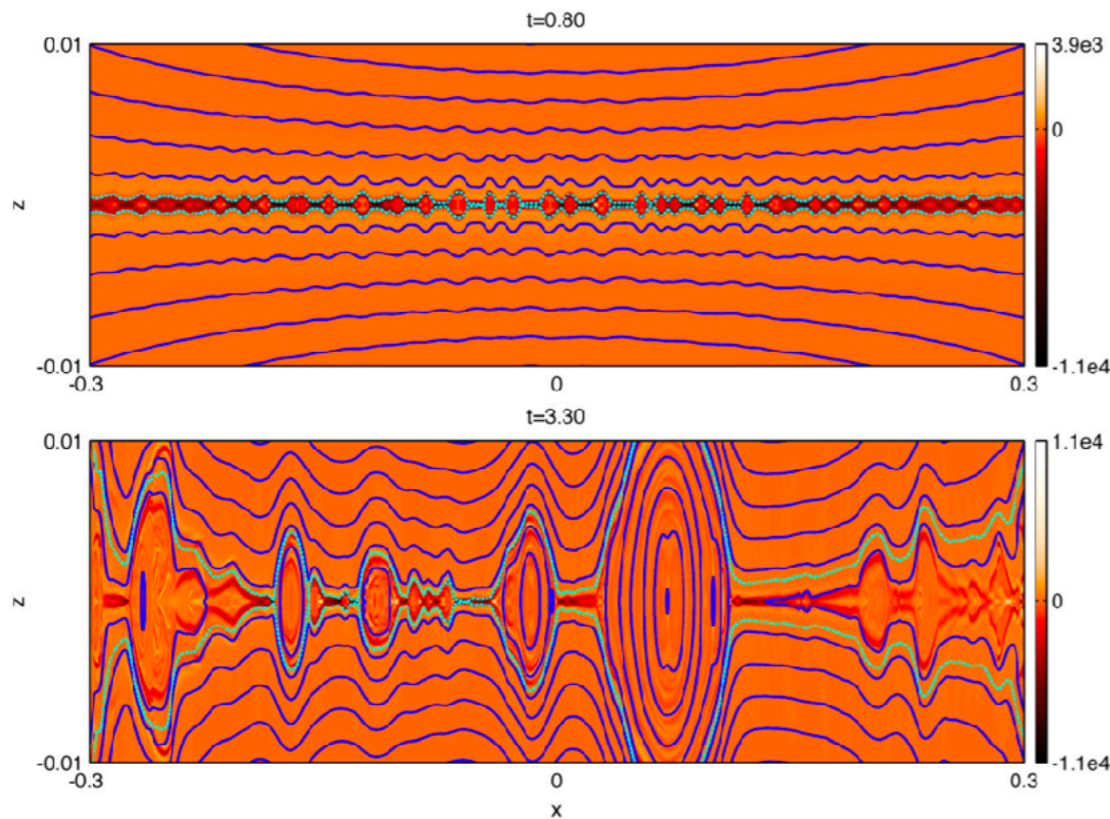
(Le+, PoP 2018)

- Self-generated turbulence in 3D sheet does **NOT** change the rate much~

Backup slides

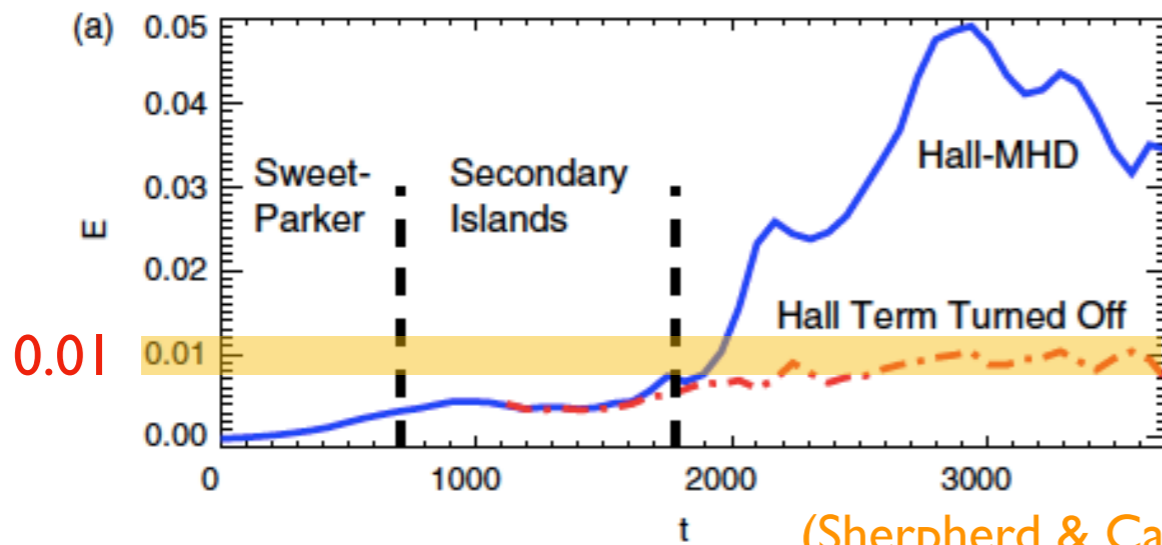
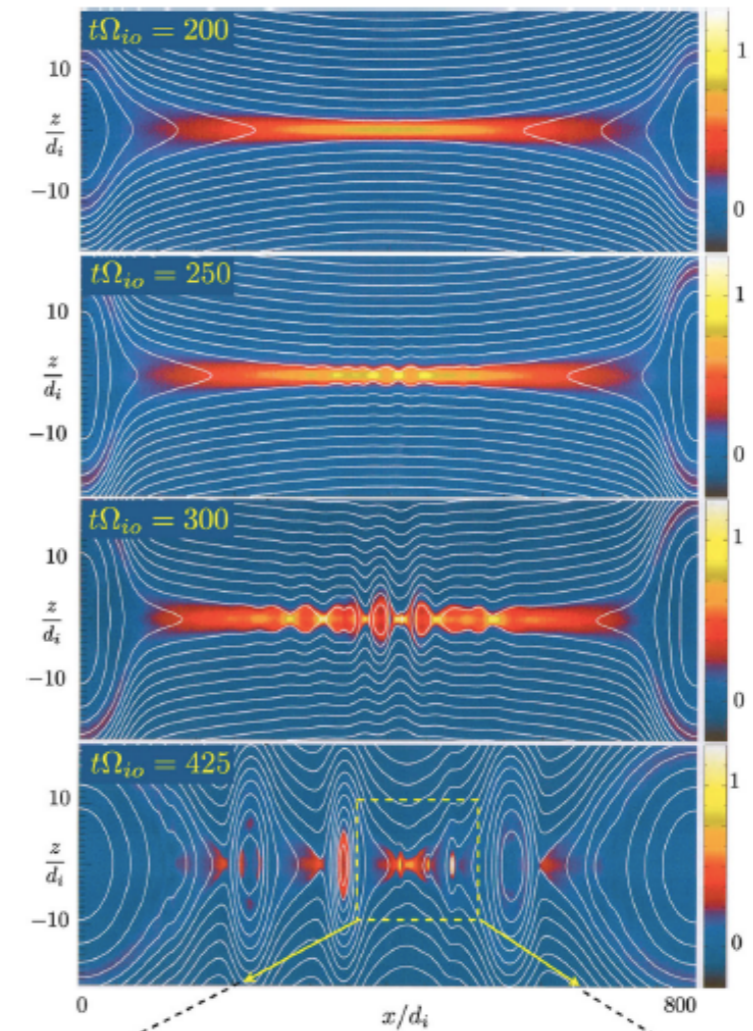
Plasmoids (i.e., secondary tearing modes)

resistive-MHD when η is very small

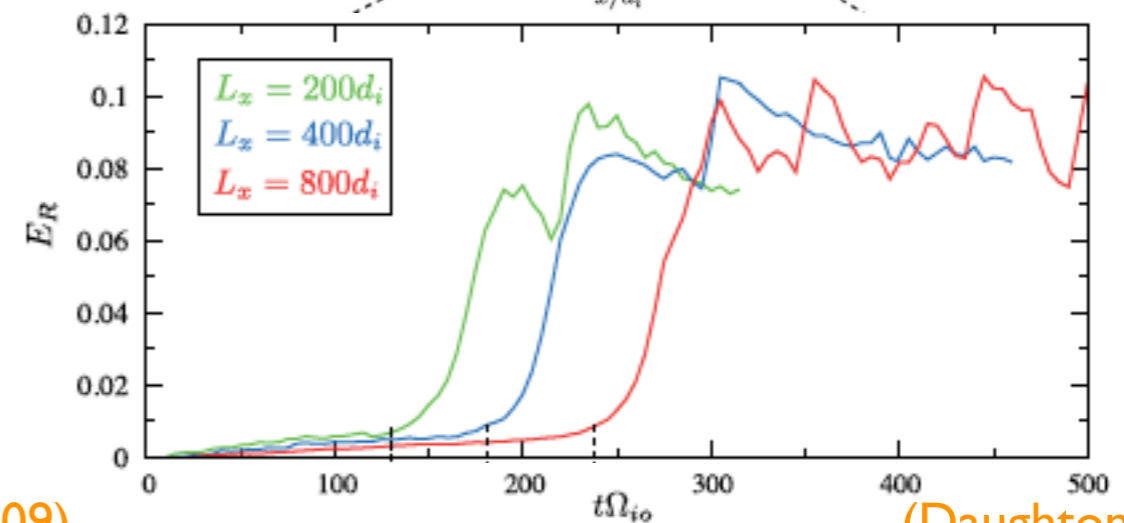


(Y.-M. Huang+ 2010, Loureiro+ 2007...)

PIC with collisions



(Sherpherd & Cassak+ 09)

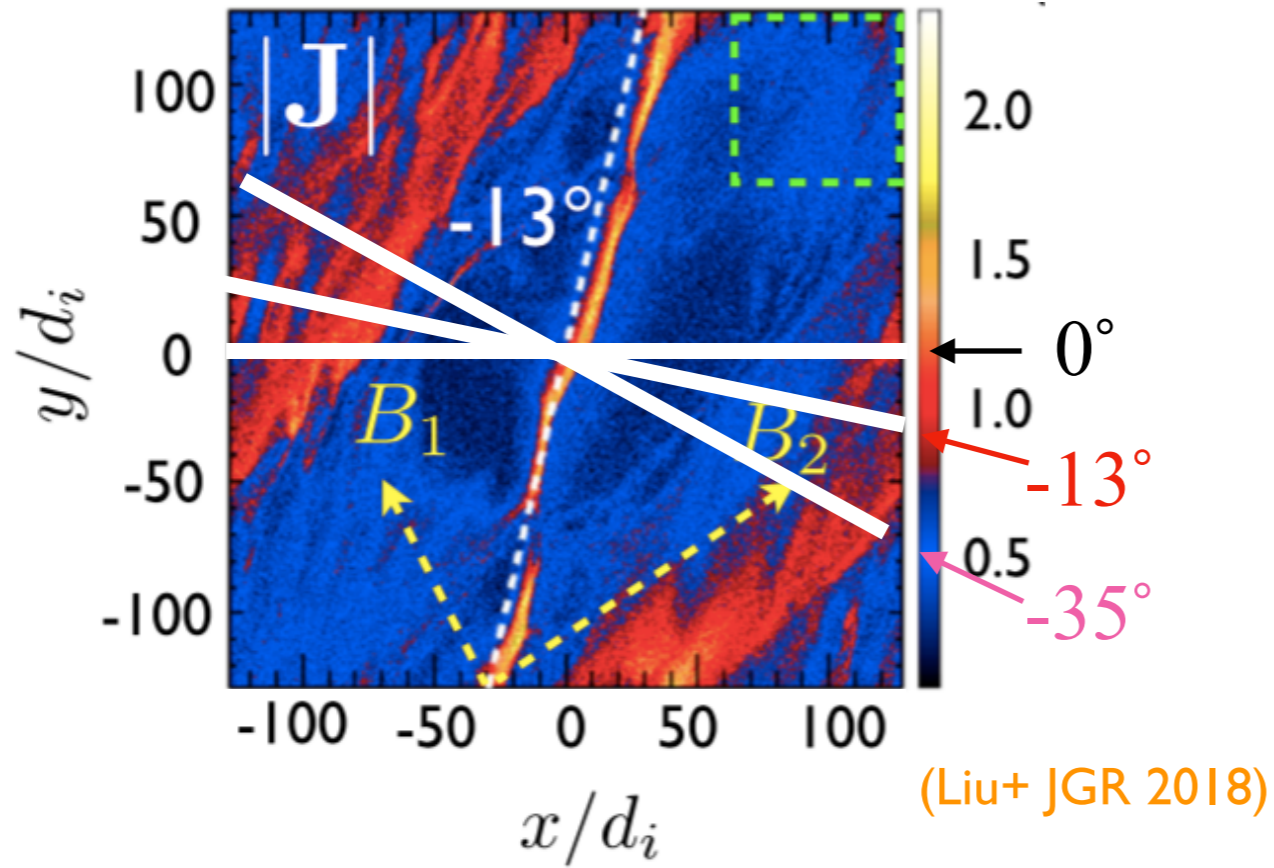


(Daughton+ 09)

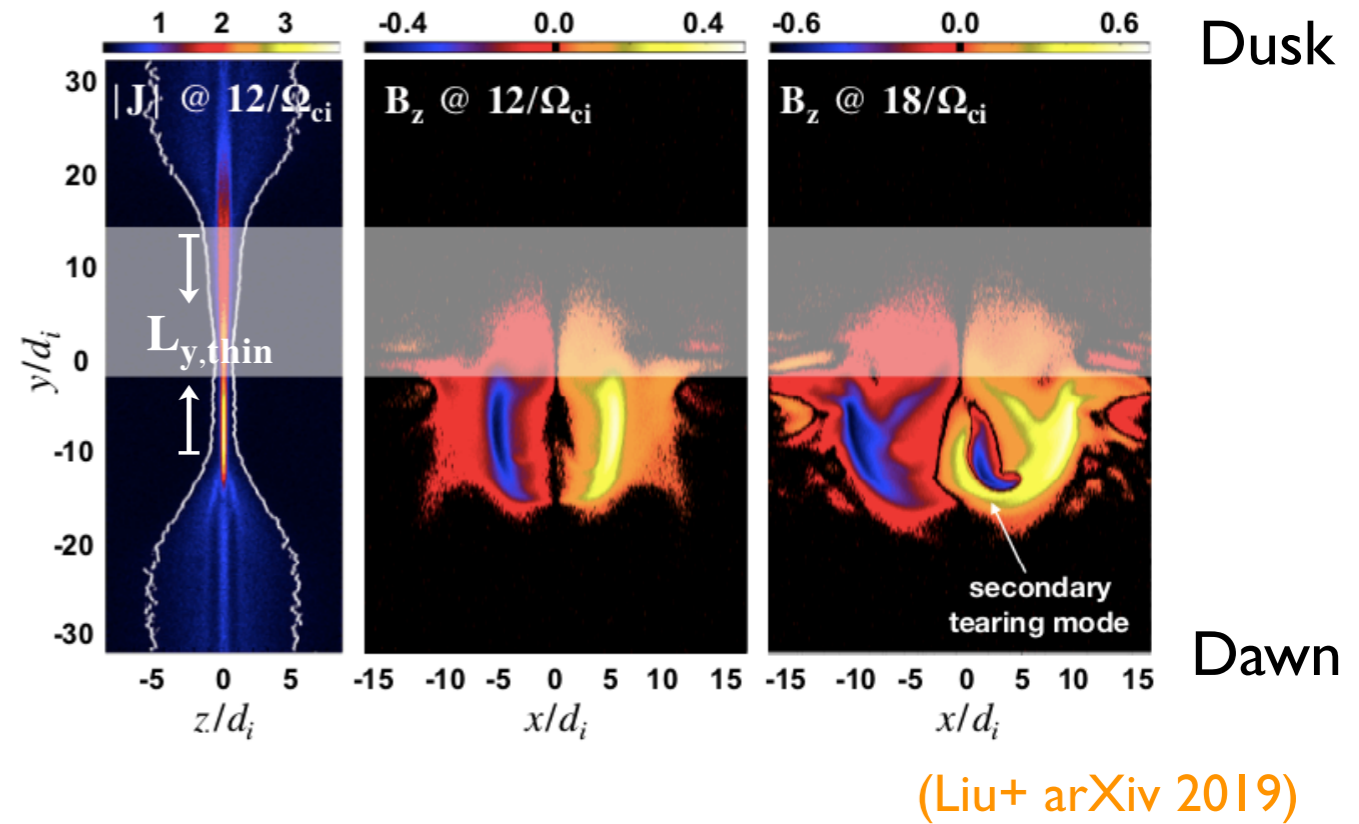
- (my opinion) Tearing may provides the localization, enhancing the rate, but cannot explain the fast rate value $\sim O(0.1)$.

3D nature

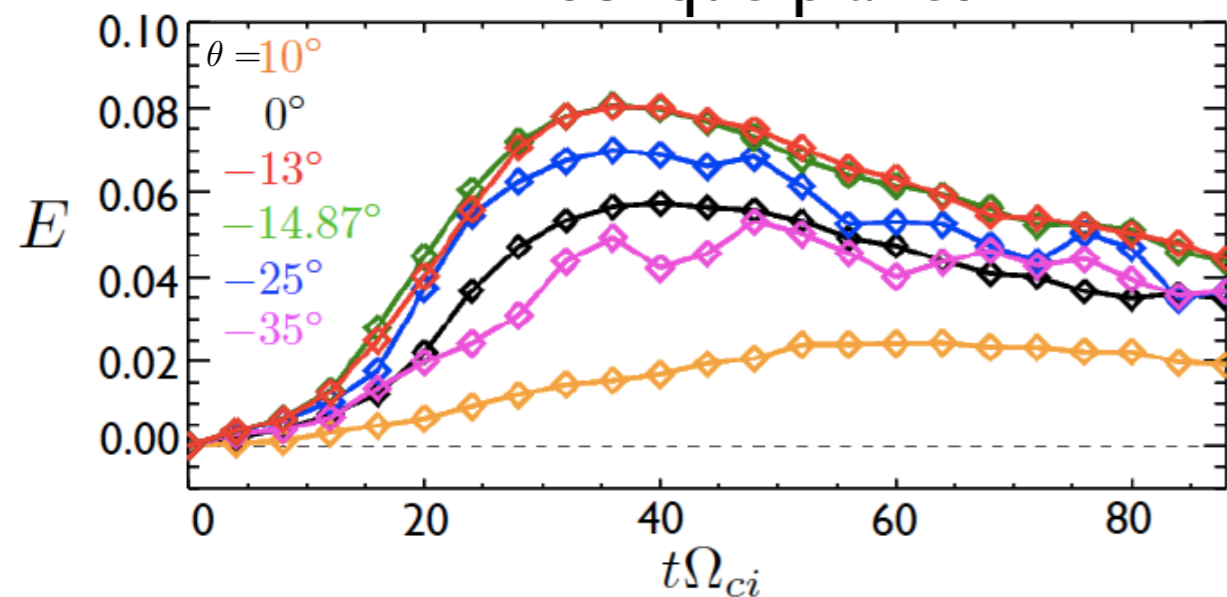
X-line orientation



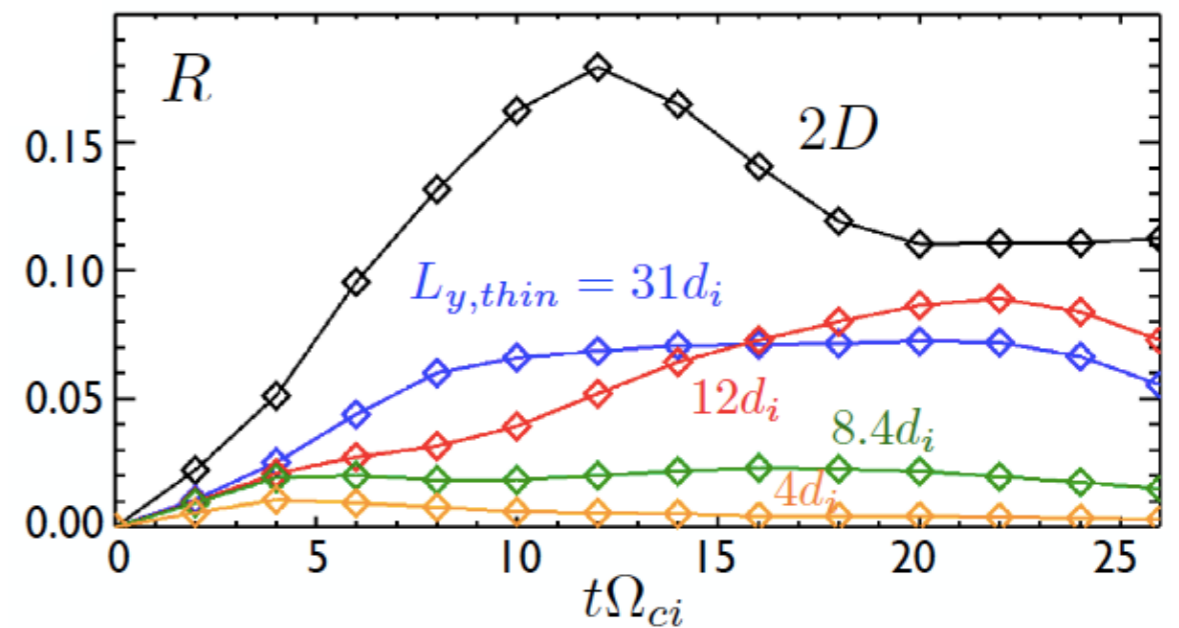
shortest possible x-line extent?



2D oblique planes

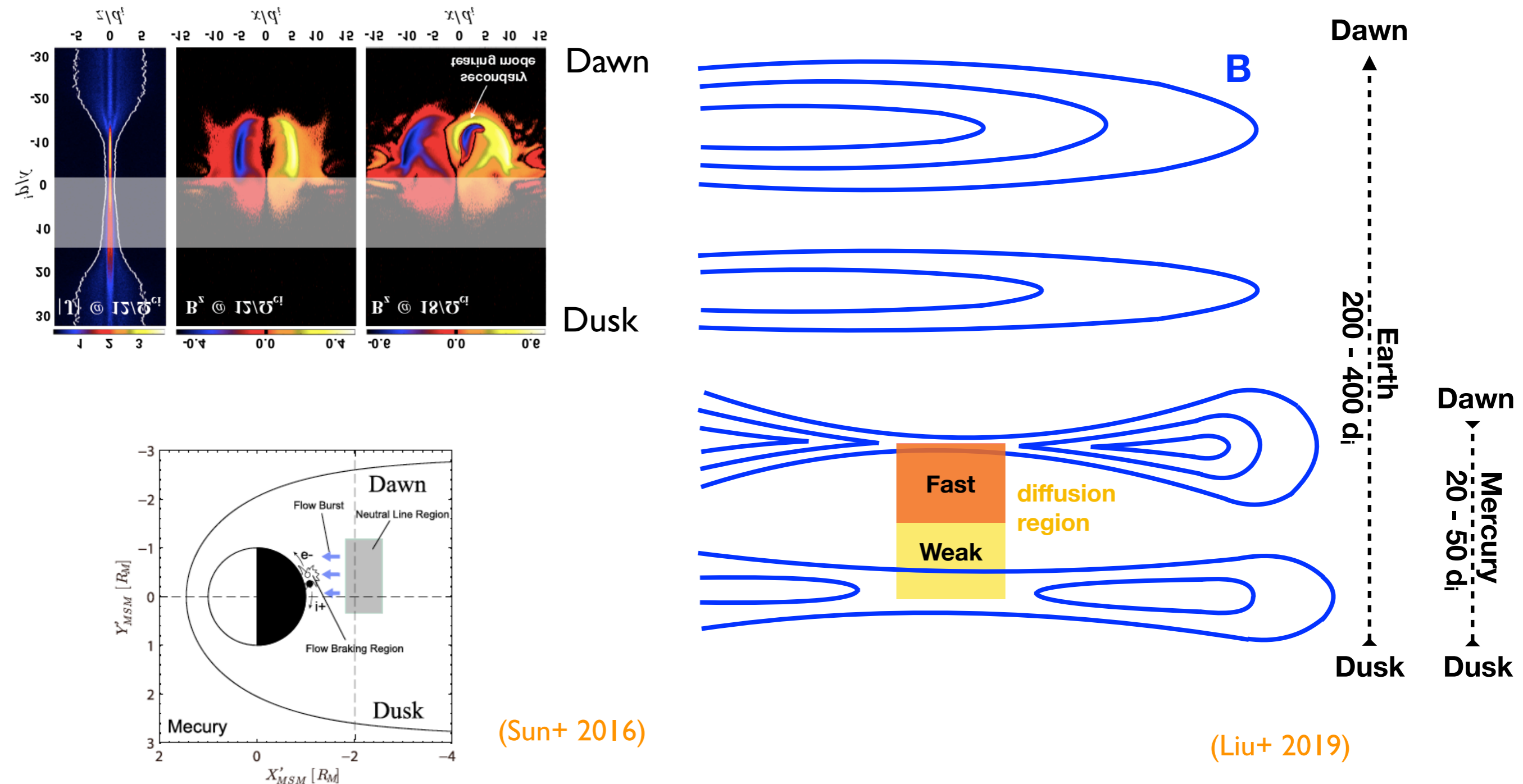


- The system tries to maximize the rate.?



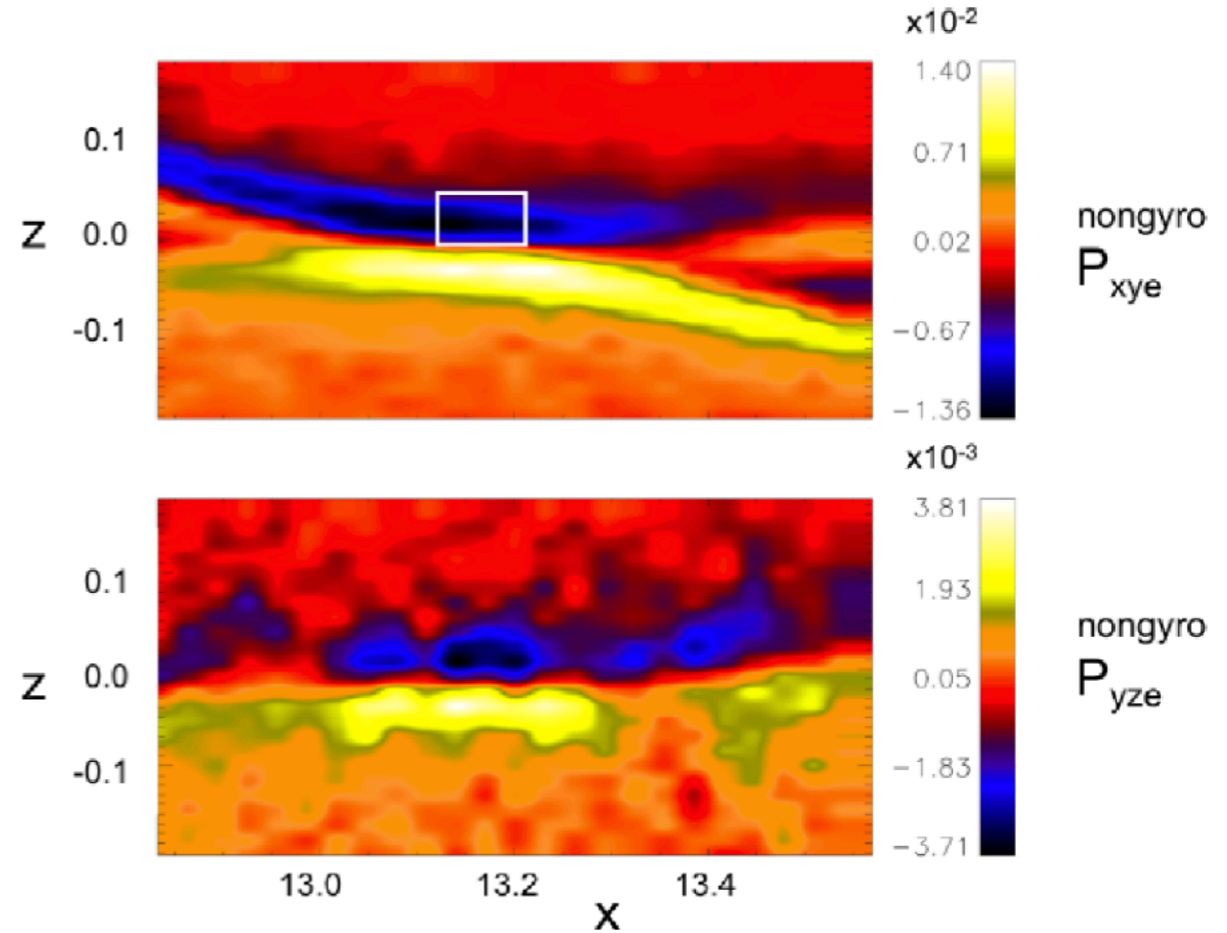
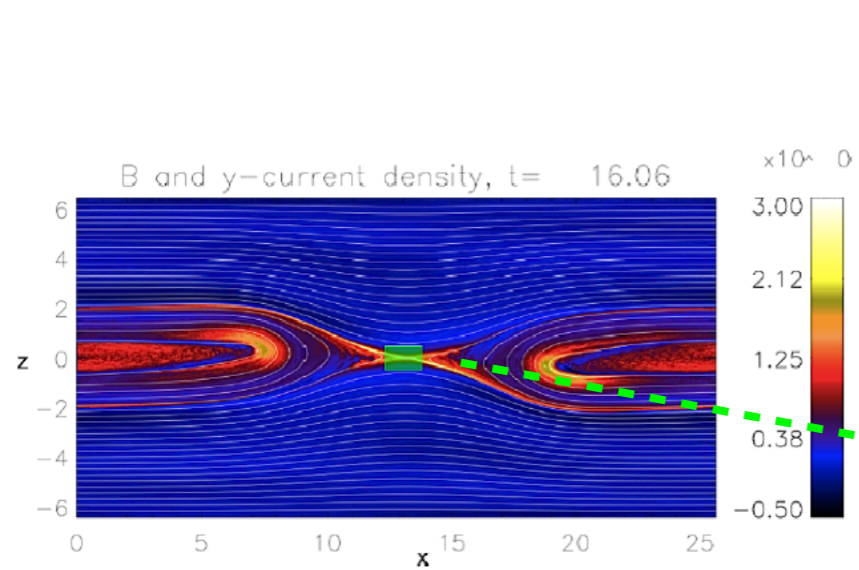
- Rate drops when the x-line extent is too short ($< 10 d_i$) — shortest possible BBF!?
- Can explain the dawn-dusk asymmetry @ Mercury!

Explanation of the opposite dawn-dusk asymmetry at Earth & Mercury's magnetotails



- An argument based on the dawn-ward flux transport & reconnection physics.

DivPe and the role of reconnection electric field



$$E_y = -\frac{1}{n_e e} \left(\frac{\partial P_{xye}}{\partial x} + \frac{\partial P_{yze}}{\partial z} \right)$$

$$E \approx \frac{1}{2en} r_L^2 \frac{\partial v_x}{\partial x} \nabla^2 (mnv_y)$$

(Hesse+ Space Sci. Rev. 2011,
tested by R. Nakamura 2018)

◆ Accelerates
the current carriers

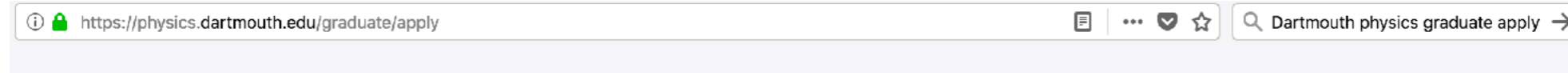
$$-\frac{\partial}{\partial t} en_e v_{ey} = \frac{e^2 n_e}{m_e} E_y + \frac{e^2 n_e}{m_e} v_{ez} B_x - \frac{e^2 n_e}{m_e} v_{ex} B_z + \frac{e}{m_e} \left(\frac{\partial P_{yze}}{\partial z} + \frac{\partial P_{xye}}{\partial x} \right) + e \nabla \cdot n_e \vec{v}_e v_{ey}$$

◆ Heats
the sheet plasma

$$\frac{\partial p}{\partial t} = -\nabla \cdot (\vec{v}p) - \frac{2}{3} \sum_l P_{ll} \frac{\partial}{\partial x_l} v_l - \frac{1}{3} \sum_{l,i} \frac{\partial}{\partial x_i} Q_{lli} - \frac{2}{3} \sum_{\substack{l,i \\ l \neq i}} P_{li} \frac{\partial}{\partial x_i} v_l$$

(Hesse+ PoP 2018)

The future needs new bloods! i.e., I need students & postdocs....

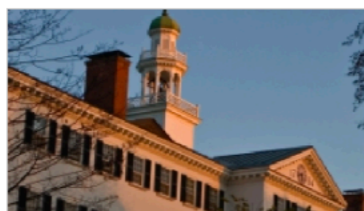


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Apply

ADMISSION REQUIREMENTS

It is expected that all incoming students will have a solid foundation in physics at the B.A. or B.S. level. Exceptions may be made for individual students with the understanding that they remedy any deficiency within the first year. Admission to the program is based on the applicant's academic record, letters of recommendation, GRE scores (general and advanced exams are required), and statement of goals. The minimum acceptable GRE score is 1200 (307), combined verbal plus quantitative sections. Foreign students must also demonstrate proficiency in written and spoken English. You may test with either ETS or the International English Language Testing System (IELTS). Minimum acceptable scores: IELTS Band score of 7.0; TWE score of 4.5, and a TOEFL score of 600 [paper-based], 250 [computer-based], or 100 [internet-based]. Official test scores must be submitted by the testing agency.

HOW TO APPLY

The application deadline is January 15.

Only electronic applications and recommendation letters will be accepted.

- Complete the online [application](#). The application fee is \$50.00.
- Arrange to have three of your instructors or research supervisors electronically submit letters of recommendation for you using the on-line application system.
- Have a complete transcript of your undergraduate record mailed directly from your university or college to:

Graduate Office
Dartmouth College
37 Dewey Field Road, Suite 6062
Hanover, NH 03755-1419